Sample Question Paper (Solved)–2025

(Issued by Central Board of Secondary Education, New Delhi)

CLASS—10th

MATHEMATICS (STANDARD)

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instructions :

Read the following instructions carefully and follow them.

1. *This question paper contains 38 questions.*
2. *This Question-paper is divided into **5** Sections – A, B, C, D, E.*
3. ***In Section A,** Questions no. 1-18 are multiple choice questions (MCQs) and questions no. 19 and 20 are Assertion-Reason based questions of 1 mark each.*
4. ***In Section B,** Questions no. 21-25 are very short answer (VSA) type questions, carrying **02** marks each.*
5. ***In Section C,** Questions no. 26-31 are short answer (SA) type questions, carrying **03** marks each.*
6. ***In Section D**, Questions no. 32-35 are long answer (LA) type questions, carrying **05** marks each.*
7. ***In Section E**, Questions no. 36-38 are case study based questions, carrying **04** marks each with sub-parts of the values of 1, 1 and 2 marks each respectively.*
8. *All questions are compulsory. However, an Internal choice in **2** questions of Section B, 2 Questions of Section C and **2** Questions of Section D has been provided. An internal choice has been provided in all the **2** marks questions of Section E.*
9. *Draw neat and clean figures wherever required.*
10. *Take $\pi = 22/7$ wherever required if not stated.*
11. *Use of calculators is not allowed.*

Section-A

Section A consists of 20 questions of 1 mark each.

Q. 1. The graph of a quadratic polynomial p (x) passes through the points (– 6, 0), (0, – 30), (4, – 20) and (6, 0). The zeroes of the polynomial are : **1**

(A) – 6, 0 (B) 4, 6

(C) – 30, – 20 (D) – 6, 6.

Ans. (D) Because the graph of quadratic polynomial $p(x)$ touches the x axis at – 6 and 6.

Q. 2. The value of k for which the system of equations 3x – ky = 7 and 6x + 10y = 3 is inconsistent is : **1**

(A) – 10 (B) – 5

(C) 5 (D) 7.

Ans. (B) The system of equations is inconsistent if

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$$\therefore \quad \frac{3}{6} = \frac{-k}{10} \neq \frac{7}{3} \Rightarrow K = -5.$$

Q. 3. Which of the following statement is not true ? **1**

(A) A number of secants can be drawn at any point on the circle.

(B) Only one tangent can be drawn at any point on a circle.

(C) A chord is a line segment joining two points on the circle.

(D) From a point inside a circle only two tangents can be drawn.

Ans. (D) From a point inside a circle only two tangents can be drawn.

Q. 4. In nth term of an A.P. is 7n – 4 then the common difference of the A.P. is : **1**

(A) 7 (B) 7n

(C) – 4 (D) 4

Ans. (A) Here $a_n = 7n - 4$

$\therefore \quad a_1 = 7(1) - 4 = 7 - 4 = 3,$

$a_2 = 7(2) - 4 = 14 - 4 = 10$

Now $d = a_2 - a_1 = 10 - 3 = 7.$

Q. 5. The radius of the base of a right circular cone and the radius of a sphere are

each 5 cm in length. If the volume of the cone is equal to the volume of the sphere then the height of the cone is : **1**

(A) 5 cm (B) 20 cm

(C) 10 cm (D) 4 cm.

Ans. (B) Volume of cone

$$= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi \times (5)^2 \times h = \frac{25\pi}{3}h$$

Volume of sphere $= \frac{4}{3}\pi r^3 = \frac{4}{3}\pi \times (5)^3 = \frac{500\pi}{3}$

Since volume of cone = Volume of sphere

$$\therefore \frac{25\pi}{3}h = \frac{500\pi}{3} \Rightarrow h = \frac{500\pi}{3} \times \frac{3}{25\pi} = 20 \text{ cm.}$$

Q. 6. If tan θ = $\frac{5}{2}$ then $\frac{4\sin\theta + \cos\theta}{4\sin\theta - \cos\theta}$ is equal to : **1**

(A) $\frac{11}{9}$ (B) $\frac{3}{2}$

(C) $\frac{9}{11}$ (D) 4.

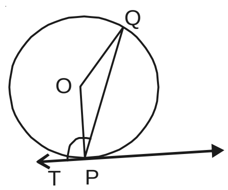
Ans. (A) Here tan $\theta = \frac{5}{2}$

$$\frac{4\sin\theta + \cos\theta}{4\sin\theta - \cos\theta} = \frac{4\tan\theta + 1}{4\tan\theta - 1}$$

[Dividing both numerator and denominator by cos θ]

$$= \frac{4 \times \frac{5}{2} + 1}{4 \times \frac{5}{2} - 1} = \frac{10+1}{10-1} = \frac{11}{9}$$

Q. 7. In the given figure, a tangent has been drawn at a point P on the circle centred at O.



If ∠TPQ = 110° then ∠POQ is equal to : **1**

(A) 110° (B) 70°

(C) 140° (D) 55°

Ans. (C) We know that a tangent is perpendicular to the radius at the point of contact

$\therefore$ ∠OPT = 90°

Now ∠TPQ = 110° (given)

$\therefore$ ∠OPQ = ∠TPQ – ∠OPT

= 110° – 90° = 20°

OP = OQ (radii of circle)

$\therefore$ ∠OQP = ∠OPQ = 20°

Now ∠POQ + ∠OQP + ∠OPQ = 180°

∠POQ + 20° + 20° = 180°

$\Rightarrow$ ∠POQ = 180° – 40° = 140°.

Q. 8. A quadratic polynomial having zeroes – $\sqrt{\frac{5}{2}}$ and $\sqrt{\frac{5}{2}}$ is : **1**

(A) $x^2 - 5\sqrt{2}\,x + 1$ (B) $8x^2 - 20$

(C) $15x^2 - 6$ (D) $x^2 - 2\sqrt{5}\,x - 1$.

Ans. (B) Sum of zeroes $= -\sqrt{\frac{5}{2}} + \sqrt{\frac{5}{2}} = 0$

Product of zeroes $= -\sqrt{\frac{5}{2}} \times \sqrt{\frac{5}{2}} = -\frac{5}{2}$

Required polynomial is :

$$x^2 - (0)\,x + \left(-\frac{5}{2}\right) = 0 \Rightarrow x^2 - \frac{5}{2} = 0$$

$\Rightarrow 2x^2 = 5 \Rightarrow 8x^2 = 20.$

Q. 9. Consider the frequency distribution of 45 observations. **1**

Class	**0–10**	**10–20**	**20–30**	**30–40**	**40–50**
Frequency	**5**	**9**	**15**	**10**	**6**

The upper limit of median class is :

(A) 20 (B) 10

(C) 30 (D) 40.

Ans. (C)

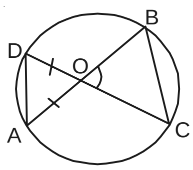
Class	Frequency	Cumulative Frequency
0 – 10	5	5
10 – 20	9	14
20 – 30	15	29
30 – 40	10	39
40 – 50	6	45

$$\therefore \frac{N}{2} = \frac{45}{2} = 22.5$$

Since the cumulative frequency just greater than 22.5 is 29.

So 20–30 is median class and upper limit of median class is 30.

Q. 10. O is the point of intersection of two chords AB and CD of a circle. **1**



If ∠BOC = 80° and OA = OD then ΔODA and ΔOBC are :

(A) equilateral and similar

(B) isosceles and similar

(C) isosceles but not similar

(D) not similar.

Ans. (B) AB and CD are diameter of circle.

∴ AB = CD

Also OA = OD (given)

∴ AB – OA = CD – OD ⇒ OB = OC

So ΔODA and ΔOBC are isosceles triangles.

In ΔODA and ΔOBC, one angle of a triangle is equal to one angle of the other triangle and the sides including these angles are proportional.

∴ ΔODA ~ ΔOBC

Q. 11. The roots of the quadratic equation are $x^2 + x - 1 = 0$ are : **1**

(A) irrational and distinct

(B) not real

(C) rational and distinct

(D) real and equal.

Ans. (A) $D = b^2 - 4ac$

$= (1)^2 - 4\,(1)\,(-1) = 1 + 4$

$= 5 > 0$

∴ Roots are real and distinct.

But D is not a perfect square.

So roots are irrational and distinct.

Q. 12. If $\theta = 30°$ then value of 3 tan θ is : **1**

(A) 1 (B) $\frac{1}{\sqrt{3}}$

(C) $\frac{3}{\sqrt{3}}$ (D) not defined.

Ans. (C) $3\tan\theta = 3\tan 30° = 3 \times \frac{1}{\sqrt{3}} = \frac{3}{\sqrt{3}}$

Q. 13. The volume of a solid hemisphere is $\frac{396}{7}$ cm^3. The total surface area of the solid hemisphere (in sq.cm) is : **1**

(A) $\frac{396}{7}$ (B) $\frac{594}{7}$

(C) $\frac{549}{7}$ (D) $\frac{604}{7}$

Ans. (B) $\frac{2}{3}\pi r^3 = \frac{396}{7} \Rightarrow \frac{2}{3} \times \frac{22}{7} \times r^3 = \frac{396}{7}$

$\Rightarrow \quad r^3 = \frac{396}{7} \times \frac{2}{2} \times \frac{7}{22}$

$\Rightarrow \quad r^3 = 27 \Rightarrow r = 3$ cm

Total surface area of hemisphere

$= 3\pi r^2 = 3 \times \frac{22}{7} \times (3)^2 = \frac{594}{7}\text{cm}^2.$

Q. 14. In a bag containing 24 balls, 4 are blue, 11 are green and the rest are white. One ball is drawn at random. The probability that drawn ball is white in colour is : **1**

(A) $\frac{1}{6}$ (B) $\frac{3}{8}$

(C) $\frac{11}{24}$ (D) $\frac{5}{8}$

Ans. (B) Total balls = 24

Number of blue balls = 4

Number of green balls = 11

Number of white balls = 24 – (11 + 4) = 9

∴ P (white ball) = $\frac{9}{24} = \frac{3}{8}$.

Q. 15. The point on the x-axis nearest to the point (–4, –5) is : **1**

(A) (0, 0) (B) (–4, 0)

(C) (–5, 0) (D) ($\sqrt{41}$, 0).

Ans. (B) All the points be on x-axis so the perpendicular distance of ponit (– 4, 5) from these points is shortest. So point (– 4, 0) is nearest to point (– 4, 5).

Q. 16. Which of the following gives the middle most observation of the data ? **1**

(A) Median (B) Mean

(C) Range (D) Mode.

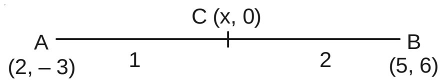
Ans. (A) Median

Q. 17. A Point on the x-axis divides the line segment joining the points A (2, –3) and B (5, 6) in the ratio 1 : 2. The point is : **1**

(A) (4, 0) (B) $\left(\frac{7}{2}, \frac{3}{2}\right)$

(C) (3, 0) (D) (0, 3).

Ans. (C) $\therefore x = \frac{2(2)+1(5)}{1+2} = \frac{4+5}{3} = 3$



Q. 18. A card is drawn from a well shuffled deck of playing cards. The probability of getting red face card is 1

(A) $\frac{3}{13}$ (B) $\frac{1}{2}$

(C) $\frac{3}{52}$ (D) $\frac{3}{26}$.

Ans. (D) Total number of outcomes = 52

Favourable outcomes = 6

$\therefore$ P (red face card) $= \frac{6}{52} = \frac{3}{26}$.

Direction : In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option :

(A) Both assertion (A) and reason (R) are true, and reason (R) is the correct explanation of assertion (A).

(B) Both assertion (A) and reason (R) are true, and reason (R) is not the correct explanation of assertion (A).

(C) Assertion (A) is true but reason (R) is false.

(D) Assertion (A) is false but reason (R) is true.

Q. 19. Assertion (A) : HCF of any two consecutive even natural number is always 2.

Reason (R) : Even natural numbers are divisible by 2. 1

Ans. Both Assertion (A) and Reason (R). Both are true but Reason (R) is not the correct explanation of Assertion (A).

Q. 20. Assertion (A) : If the radius of sector of a circle is reduced to its half and angle is doubled then the perimeter of the sector remains the same.

Reason (R) : The length of the arc subtending angle θ at the centre of a circle of radius $r = \frac{\pi r\theta}{180}$ 1

Ans. (D) Assertion (A) is false because length of arc in both the cases is same but the perimeter decreases. Reason (R) is true.

Section-B

Section B consists of 5 questions of 2 marks each.

Q. 21. (A) Find the H.C.F. and L.C.M. of 480 and 720 using the Prime factorisation method. 2

Sol.

2	480
2	240
2	120
2	60
2	30
3	15
5	5
	1

2	720
2	360
2	180
2	90
3	45
3	15
5	15
	5

Here $480 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 5$

$= 2^5 \times 3 \times 5$

$720 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$= 2^4 \times 3^2 \times 5$

HCF (480, 720) $= 2^4 \times 3 \times 5 = 240$

LCM (480, 720) $= 2^5 \times 3^2 \times 5 = 1440$.

Or

(B) The H.C.F. of 85 and 238 is expressible in the form 85 m – 238. Find the value of m. 2

Sol.

5	85
17	17
	1

2	238
7	119
17	17
	1

Here $85 = 5 \times 17$

$238 = 2 \times 7 \times 17$

HCF (85, 238) = 17

$\therefore$ $85m - 238 = 17 \Rightarrow 85m = 255 \Rightarrow m = 3$

Q. 22. (A) Two dice are rolled together bearing numbers 4, 6, 7, 9, 11, 12. Find the probability that the product of numbers obtained in an odd number 2

Sol. Total number of outcomes $= 6 \times 6 = 36$

Favourable outcomes are (7, 7), (7, 9), (7, 11), (9, 7), (9, 9), (9, 11), (11, 7) (11, 9), (11, 11)

Number of favourable outcomes = 9

P (product of numbers is odd) $= \frac{9}{36} = \frac{1}{4}$.

Or

(B) How many positive three digit integers have the hundredths digit 8 and unit's digit 5 ?

Find the probability of selecting one such number out of all three digit numbers.

Sol. Total number of three digit numbers = 900

Numbers with hundredth digit 8 and unit's digit 5 are 805, 815, 825, 835, 845, 855, 865, 875, 885, 895

Number of favourable outcomes = 10

P (hundredth digit 8 and unit's digit 5) = $\frac{10}{900} = \frac{1}{90}$.

Q. 23. Evaluate : $\frac{\mathbf{2\sin^2 60^o - \tan^2 30^o}}{\mathbf{\sec^2 45^o}}$ **2**

Sol. $$\frac{2\sin^2 60^o - \tan^2 30^o}{\sec^2 45^o} = \frac{2\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2}{(\sqrt{2})^2}$$

$$= \frac{2\times\frac{3}{4}-\frac{1}{3}}{2} = \frac{\frac{3}{2}-\frac{1}{3}}{2} = \frac{\frac{9-2}{6}}{2} = \frac{7}{6}\times\frac{1}{2} = \frac{7}{12}$$

Q. 24. Find the point (s) on the x-axis which is at a distance of $\sqrt{41}$ units from the point (8, –5). **2**

Sol. Let point on x-axis be $(x, 0)$

$\therefore \sqrt{(8-x)^2+(-5-0)^2} = \sqrt{41}$

$\Rightarrow \sqrt{(8-x)^2+25} = \sqrt{41} \Rightarrow (8-x)^2 = 41-25$

$\Rightarrow (8-x)^2 = 16 \Rightarrow (8-x) = \pm 4$

$\Rightarrow x = 4, 12$

$\therefore$Points on x-axis are (4, 0) and (12, 0)

Q. 25. Show that the points A (–5, 6), B (3, 0) and C (9, 8) are the vertices of an isosceles triangle. **2**

Sol. $AB = \sqrt{(3+5)^2+(0-6)^2}$

$= \sqrt{64+36} = \sqrt{100} = 10$

$BC = \sqrt{(9-3)^2+(8-0)^2}$

$= \sqrt{36+64} = \sqrt{100} = 10$

$CA = \sqrt{(-5-9)^2+(6-8)^2}$

$= \sqrt{196+4} = \sqrt{200} = 10\sqrt{2}$

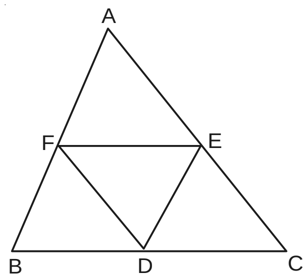
$\therefore AB = BC$

So ΔABC is an isosceles triangle.

Section-C

Section C consists of 6 questions of 3 marks each.

Q. 26. (A) In ΔABC,D, E and F are midpoints of BC, CA and AB respectively. Prove that Δ FBD ~ Δ DEF and Δ DEF ~ Δ ABC. **3**



Sol. It is given that D, E and F are mid point of BC, CA and AB respectively

$\therefore$ EF ∥ BC, DF ∥ AC and DE ∥ AB

$\therefore$ BDEF is a parallelogram

$\angle DEF = \angle B$ and $\angle DFE = \angle BDF$

$\therefore \Delta FBD \sim \Delta DEF$

Also DCEF is a parallelogram

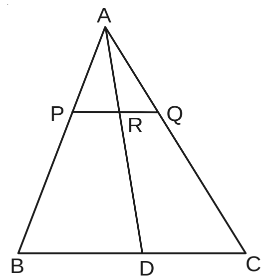
$\therefore \angle DFE = \angle C$ and $\angle DEF = \angle B$ (proved above)

$\therefore \Delta DEF \sim \Delta ABC$

Or

(B) In ΔABC, P and Q are points on AB and AC respectively such that PQ is parallel to BC.

Prove that the median AD drawn from A and BC bisects PQ.



Sol. It is given that PQ ∥ BC

$\therefore \Delta APR \sim \Delta ABD$

$$\frac{AP}{AB} = \frac{PR}{BD} \quad ....(i)$$

Also $\Delta AQR \sim \Delta ACD$

$$\frac{AQ}{AC} = \frac{RQ}{DC} \quad ....(ii)$$

Now $$\frac{AP}{AB} = \frac{AQ}{AC} \quad ....(iii)$$

From equations (*i*), (*ii*) and (*iii*), we get

$$\frac{PR}{BD} = \frac{RQ}{DC}$$

But $BD = DC$ (given)

$\therefore$ $PR = RQ \Rightarrow$ AD bisects PQ.

Q. 27. The sum of two numbers is 18 and the sum of their reciprocals is 9/40. Find the numbers. **3**

Sol. Let numbers be x and $(18 - x)$

$$\therefore \quad \frac{1}{x} + \frac{1}{(18-x)} = \frac{9}{40}$$

$$\Rightarrow \quad \frac{18 - x + x}{x(18-x)} = \frac{9}{40}$$

$$\Rightarrow \quad 18 \times 40 = 9 \times (18 - x)$$

$$\Rightarrow \quad \frac{18}{18x - x^2} = \frac{9}{40}$$

Cross Multiplying

$$9(18x - x^2) = 18 \times 40$$

Dividing both sides by 9

$$18x - x^2 = 2 \times 40 \Rightarrow 18x - x^2 = 80$$

$$\Rightarrow \quad x^2 - 18x + 80 = 0$$

$$x^2 - 10x - 8x + 80 = 0$$

$$x(x - 10) - 8(x - 10) = 0$$

$$(x - 10)(x - 8) = 0$$

$$x - 10 = 0 \text{ or } x - 8 = 0$$

$$x = 10, \text{ or } x = 8$$

$$x = 10, 8$$

Thus required numbers are 8 and 10.

Q. 28. If α and β are zeroes of a polynomial $6x^2 - 5x + 1$ then form a quadratic polynomial whose zeroes are α^2 and β^2. **3**

Sol. The given polynomial is $6x^2 - 5x + 1$

$$\Rightarrow \quad \alpha + \beta = \frac{5}{6} \text{ and } \alpha\beta = \frac{1}{6}$$

Now $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$$= \left(\frac{5}{6}\right)^2 - 2\left(\frac{1}{6}\right) = \frac{25}{36} - \frac{1}{3}$$

$$= \frac{25 - 12}{36} = \frac{13}{36}$$

$$\alpha^2\beta^2 = \left(\frac{1}{6}\right)^2 = \frac{1}{36}$$

$\therefore$Required polynomial is $x^2 - \frac{13}{36}x + \frac{1}{36}$

$$= 36x^2 - 13x + 1.$$

Q. 29. If $\cos\theta + \sin\theta = 1$, then prove that $\cos\theta - \sin\theta = \pm 1$. **3**

Sol. $(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2 = \cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta$

$$= 1 + 1 = 2$$

$\Rightarrow (1)^2 + (\cos\theta - \sin\theta)^2 = 2$

$\Rightarrow (\cos\theta - \sin\theta)^2 = 2 - 1 \Rightarrow (\cos\theta - \sin\theta)^2 = 1 \Rightarrow \cos\theta - \sin\theta = \pm 1$

Q. 30. (A) The minute hand of a wall clock is 18 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes. **3**

Sol. Angle described by minute hand in 60 minutes = 360°

Angle described by minute hand in 5 minutes

$$= \frac{360}{60} \times 5 = 30°$$

Length of minute hand (r) = 18 cm

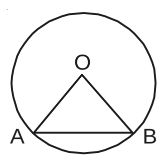
Area swept by minute hand in 35 minutes

$$= 7\left(\frac{22}{7} \times 18 \times 18 \times \frac{30}{360}\right)$$

$$= 7 \times \frac{594}{7} = 594 \text{ cm}^2.$$

Or

(B) AB is a chord of a circle centred at O such that $\angle AOB = 60°$, If OA = 14 cm then find the area of the minor segment. (take = $\sqrt{3}$ =1.73) **2**



Sol. Area of minor segment = area of sector OAB – area of ΔOAB

$$= \frac{22}{7} \times 14 \times 14 \times \frac{60}{360} - \frac{\sqrt{x}}{4} \times 14 \times 14$$

$= 102.67 - 84.77 = 17.9 \text{ cm}^2$.

Q. 31. Prove that $\sqrt{3}$ is an irrational number. **3**

Ans. Let us suppose that $\sqrt{3}$ be rational

$$\therefore \quad \sqrt{3} = \frac{a}{b} \Rightarrow a^2 = 3b^2 \quad ....(1)$$

$\Rightarrow$3 divides $a^2 \Rightarrow$ 3 divides a

Putting $a = 3$ m in equation (*i*), we get :

$$(3m)^2 = 3b^2 \Rightarrow 9m^2 = 3b^2$$

$$\Rightarrow \quad b^2 = 3m^2$$

$\Rightarrow$ 3 divides $b^2 \Rightarrow$ 3 divides b

Thus 3 is common factor of a and b

which is a contradiction to our supposition

Hence $\sqrt{3}$ is an irrational number.

Section-D

Section D consists of 4 questions of 5 marks each.

Q. 32. (A) Solve the following system of linear equations graphically. 5

x + 2y = 3, 2x – 3y + 8 = 0

Sol. Table for the line $x + 2y = 3$

x	1	– 1	3
y	1	2	0

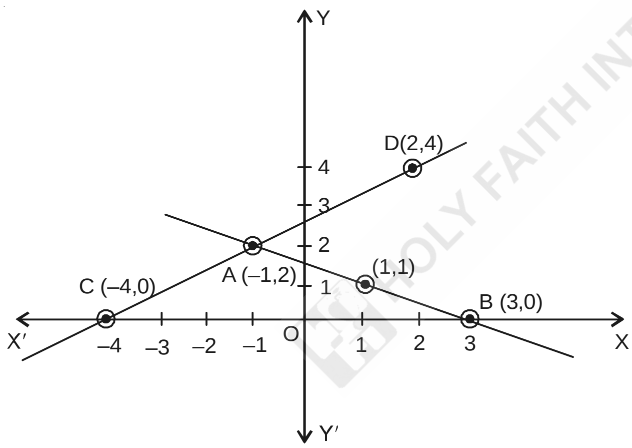
Table for the line $2x - 3y + 8 = 0$

x	– 4	– 1	2
y	0	2	4

Plot all the points of first table on a graph paper and join them to get a straight line AB. Also plot all the points of second table on the same graph paper and join them to get a straight line CD.

Both the lines AB and CD intersect at point (– 1, 2)

Thus $x = -1$ and $y = 2$ is required solution of system of equations.



Or

(B) Places A and B are 180 km apart on a highway. One car starts from A and another from B at the same time. If the car travels in the same direction at different speeds, they meet in 9 hours. If they travel towards each other with the same speeds as before, they meet in an hour. What are the speeds of the two cars ?

Sol. Let speed of car I = x km/h

and speed of car II = y km/h

Let car I starts from point A and car II starts from point B

Case I : When cars are moving in same direction

Distance travelled by car I in 9 hours = $9x$ km

Distance travelled by car II in 9 hours = $9y$ km

$\therefore \quad 9x - 9y = 180 \Rightarrow x - y = 20 \quad ....(i)$

Case II : When cars are moving in opposite directions

Distance travelled by car I in 1 hour = x km

Distance travelled by car II in 1 hour = y km

$\therefore \quad x + y = 180 \quad ....(ii)$

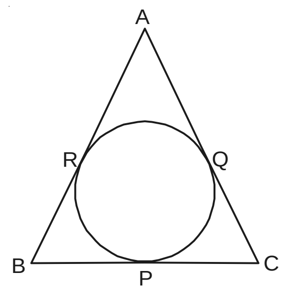
Adding equations (i) and (ii), we get

$$2x = 200 \Rightarrow x = 100 \text{ km/h}$$

Putting $x = 100$ in equation (ii), we get

$$100 + y = 180 \Rightarrow y = 80 \text{ km/h}$$

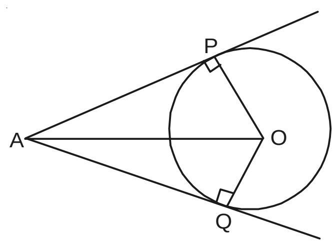
Q. 33. Prove that the lengths of tangents drawn from an external point to a circle are equal. Using above result, find the length BC of Δ ABC. Given that, a circle is inscribed in Δ ABC touching the sides AB, BC and CA at R, P and Q respectively and AB = 10 cm, AQ = 7 cm, CQ = 5 cm. 5



Sol. Given AP and AQ are two tangents from a point A to a circle having centre O.

To Prove : AP = AQ

Construction : Join OP, OQ and OA.



Proof : We know that a tangent at any point of a circle is perpendicular to the radius through the point of contact

$\therefore$ OP ⊥ AP and OQ ⊥ AQ

$\Rightarrow \quad \angle OPA = 90° \text{ and } \angle OQA = 90°$

In ΔOPA and ΔOQA

OA = OA ...(Common side)

∠OPA = ∠OQA (each 90°)

OP = OQ ...(radii of circle)

So, ΔOPA ≅ ΔOQA(by RHS congruence rule)

$\therefore$ AP = AQ (by cpct)

Now AR = AQ = 7 cm

BP = BR = AB – AR = 3 cm

CP = CQ = 5 cm

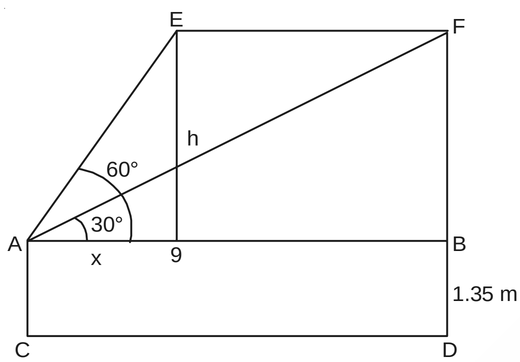
∴ BC = BP + PC = 3 + 5 = 8 cm.

Q. 34. A boy whose eye level is 1.35 m from the ground, spots a balloon moving with the wind in a horizontal line at some height from the ground. The angle of elevation of the balloon from the eyes of the boy at an instant is 60°. After 12 seconds, the angle of elevation reduces to 30°. If the speed of the wind is 3m/s then find the height of the balloon from the ground.(Use $\sqrt{3}$ = 1.73). 5

Sol. Let A be the eye level and E and F are positions of bolloon

Distance covered by balloon in 12 seconds

= 3 × 12 = 36 m



∴ EF = GB = 36 m

In ΔAGE $\frac{EG}{AG} = \tan 60° \Rightarrow \frac{h}{x} = \sqrt{3}$

⇒ $h = \sqrt{3}x$ ....(*i*)

In ΔABF $\frac{BF}{AB} = \tan 30° \Rightarrow \frac{EG}{AG + GB}$

$= \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{x+36} = \frac{1}{\sqrt{3}} \Rightarrow h = \frac{x+36}{\sqrt{3}}$ ....(*ii*)

From equations (*i*) and (*ii*), we get

$$\sqrt{3}x = \frac{x+36}{\sqrt{3}} \Rightarrow 3x = x + 36$$

⇒ $2x = 36 \Rightarrow x = 18$ m

Putting $x = 18$ in equation (1), we get

∴ $h = \sqrt{3} \times 18 = 1.73 \times 18$

$= 31.14$ m

So height of balloon from the ground = 31.14 + 1.35 = 32.49 m

Q. 35. Find the mean and median of the following data : 5

Class	**85-90**	**90-95**	**95-100**	**100-105**	**105-110**	**110-115**
frequency	**15**	**22**	**20**	**18**	**20**	**25**

Sol.

Class	x	f	$u = \frac{x-a}{h}$	fu	$c.f.$
85–90	87.5	15	– 3	– 45	15
90–95	92.5	22	– 2	– 44	37
95–100	97.5	20	– 1	– 20	57
100–105	102.5	18	0	0	85
105–110	107.5	20	1	20	95
110–115	112.5	25	2	50	120
		120		– 39	

Here $a = 102.5$ and $h = 5$

$$\bar{x} = a + h \times \frac{\Sigma fu}{\Sigma f}$$

$$\bar{x} = 102.5 + 5 \times \frac{-39}{120}$$

$$= 102.5 - 1.625 = 100.875$$

$$\frac{N}{2} = \frac{120}{2} = 60$$

∴100–105 is median class

∴ $l = 100$, $c.f. = 57$, $h = 5$

and $f = 18$

$$\text{Median} = l + \left[\frac{\frac{N}{2} - c.f.}{f}\right] \times h$$

$$= 100 + \left[\frac{60-57}{18}\right] \times 5$$

$$= 100 + \frac{3 \times 5}{18}$$

$$= 100 + 0.83 = 100.83$$

Or

The monthly expenditure on milk in 200 families of a Housing Society is given below

Monthly Expenditure (in ₹)	**1000-1500**	**1500-2000**	**2000-2500**	**2500-3000**	**3000-3500**	**3500-4000**	**4000-4500**	**4500-5000**
Number of families	**24**	**40**	**33**	**X**	**30**	**22**	**16**	**7**

Find the value of x and also find the mean expenditure.

Sol.

Monthly Expenditure (in ₹)	f	x	fx
1000–1500	24	1250	30000
1500–2000	40	1750	70000
2000–2500	33	2250z	74250
2500–3000	x	2750	77000
3000–3500	30	3250	97500
3500–4000	22	3750	82500
4000–4500	16	4250	68000
4500–5000	7	4750	33250
	172 + x		532500

Now $172 + x = 200 \Rightarrow x = 200 - 172 = 28$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{532500}{200} = 2662.5$$

Section-E

Section E consists of 3 case study questions of 4 marks each.

Q. 36. Ms. Sheela visited a store near her house and found that the glass jars are arranged one above the other in a specific pattern.

On the top layer there are 3 jars. In the next layer there are 6 jars. In the 3rd layer from the top there are 9 jars and so on till the 8th layer.

On the basis of the above situation answer the following questions.

(*i*) Write an A.P. whose terms represent the number of jars in different layer starting from top. Also, find the common difference. 1

(*ii*) Is it possible to arrange 34 jars in a layer if this pattern is continued ? Justify your answer. 1

(*iii*) (A) If there are 'n' number of rows in a layer then find the expression for finding the total number of jars in terms of n. Hence find S_8. 2

Sol. (*i*) A.P is 3, 6, 9, ... , 24

$d = 6 - 3 = 3$

(*ii*) $a_n = a + (n-1)\,d$

$\therefore$ $34 = 3 + (n-1)(3)$

$\Rightarrow$ $3n - 3 = 31$

$\Rightarrow$ $3n = 34 \Rightarrow n = \frac{34}{3} = 11\frac{1}{3}$.

Here n is not a positive integer. So it is not possible to have 34 jars in a layer if the given pattern is continued.

(*iii*) (A) $S_n = \frac{n}{2}[2a + (n-1)\,d]$

$= \frac{n}{2}[2 \times 3 + (n-1)(3)]$

$= \frac{n}{2}[6 + 3n - 3] = \frac{n}{2}[3 + 3n]$

$\therefore$ $S_8 = \frac{8}{2}[3 + 3 \times 8]$

$= 4 \times 27 = 108.$

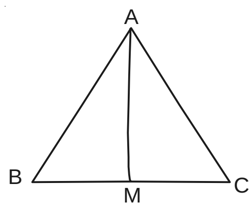
Or

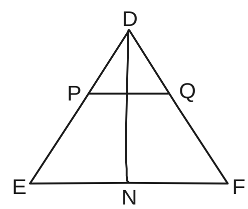
(*iii*) (B) The shopkeeper added 3 jars in each layer. How many jars are there in the 5th layer from the top ? 2

Ans. $a_5 = a + (5-1)\,d$

$= 6 + 4\,(3) = 6 + 12 = 18.$

Q. 37.





Triangle is a very popular shape used in interior designing. The picture given above shows a cabinet designed by a famous interior designer.

Here the largest triangle is represented by ΔABC and smallest one with shelf is represented by ΔDEF, PQ is parallel to EF.

(*i*) Show that Δ DPQ ~ Δ DEF 1

(*ii*) If DP = 50 cm and PE = 70 cm then find $\frac{PQ}{EF}$.

(*iii*)(A) If 2AB = 5DE and ΔABC ~ ΔDEF then show that $\frac{\text{perimeter of } \Delta ABC}{\text{perimeter of } \Delta DEF}$ is constant. 2

Sol. (*i*) Since PQ || EF

$\therefore$ $\angle DPQ = \angle DEF$ and $\angle DQP = \angle DFE$

So $\Delta DPQ \sim \Delta DEF$

(*ii*) DP = 50 cm and PE = 70 cm

$\therefore$ DE = DP + PE = 50 + 70 = 120 cm

Now $\dfrac{DP}{DE} = \dfrac{PQ}{EF} \Rightarrow \dfrac{PQ}{EF} = \dfrac{50}{120} = \dfrac{5}{12}$

(*iii*) (A) $2AB = 5DE \Rightarrow \dfrac{AB}{DE} = \dfrac{5}{2}$

Also $\Delta ABC \sim \Delta DEF$

$$\therefore \quad \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{5}{2}$$

$$\Rightarrow \quad AB = \frac{5}{2}DE,\ BC = \frac{5}{2}EF$$

and $AC = \dfrac{5}{2}DF$

Now $\dfrac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta DEF} = \dfrac{AB+BC+AC}{DE+EF+DF}$

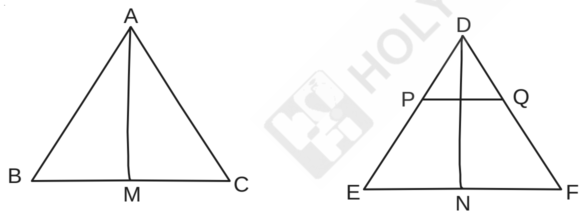
$$= \frac{\frac{5}{2}DE+\frac{5}{2}EF+\frac{5}{2}DF}{DE+EF+DF} = \frac{\frac{5}{2}(DE+EF+DF)}{DE+EF+DF}$$

$$= \frac{5}{2} = \text{constant}.$$

Or

***(iii)* (B) If AM and DN are medians of triangles ABC and DEF respectively then prove that ΔABM ~ ΔDEN.**

Sol.



$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{\frac{BC}{2}}{\frac{EF}{2}} = \frac{BM}{EN}$$

and $\angle B = \angle E$

$\therefore \Delta ABM \sim \Delta DEN$

Q. 38. Metallic silos are used by farmers for storing grains. Farmer Girdhar has decided to build a new metallic silo to store his harvested grains. It is in the shape of a cylinder mounted by a cone.

Dimensions of the conical part of a silo is as follows :

Radius of base = 1.5 m

Height = 2 m

Dimensions of the cylindrical part of a silo is as follows :

Radius of base = 1.5 m

Height = 7 m

On the basis of the above information answer the following questions.

(*i*) Calculate the slant height of the conical part of one silo. **1**

(*ii*) Find the curved surface area of the conical part of one silo. **1**

(*iii*) (A) Find the cost of metal sheet used of make the curved cylindrical part of 1 silo at the rate of ₹ 2000 per m². **2**

Sol. (*i*) $l = \sqrt{h^2 + r^2}$

$$= \sqrt{(2)^2 + (1.5)^2} = \sqrt{4 + 2.25}$$

$$= \sqrt{6.25} = 2.5 \text{ m}$$

(*ii*) Curved surface area of cone = πrl

$$= \frac{22}{7} \times 1.5 \times 2.5 = 11.78 \text{ m}^2$$

(*iii*) (A) Curved surface area of cylinder = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 1.5 \times 7 = 66 \text{ m}^2$$

Cost of metal sheet used = 66 × 2000 = ₹ 132000.

Or

(*iii*) (B) Find the total capacity of one silo to store grains. **2**

Ans. Volume of cylinder = $\pi r^2 h$

$$= \frac{22}{7} \times (1.5)^2 \times 7 = 49.5 \text{ m}^3$$

Volume of cone = $\dfrac{1}{3}\pi r^2 h$

$$= \frac{1}{3} \times \frac{22}{7} \times (1.5)^2 \times 2 = 4.71 \text{ m}^3$$

Total capacity = 49.5 + 4.71 = 54.21 m³.

MODEL ANSWERS

Holy Faith New Style Sample Paper–1

(Based on the Latest Design & Syllabus Issued by CBSE)

CLASS — 10th (CBSE)

MATHEMATICS (STANDARD)

Time Allowed : Three Hours] **[Maximum Marks : 80**

General Instructions : *Read the following instructions carefully and follow them :*

(*i*) *This question paper contains* ***38*** *questions.* ***All*** *questions are* ***compulsory.***

(*ii*) *This question paper is divided into* ***five*** *Sections–****A, B, C, D*** *and* ***E.***

(*iii*) *In* ***Section A,*** *Question numbers,* ***1*** *to* ***18*** *are multiple choice questions (MCQs) and question numbers* ***19*** *and* ***20*** *are Assertion–Reason based questions of* ***1*** *mark each.*

(*iv*) *In* ***Section B,*** *Question numbers* ***21*** *to* ***25*** *are very short answer (VSA) type questions, carrying* ***2*** *marks each.*

(*v*) *In* ***Section C,*** *Question numbers* ***26*** *to* ***31*** *are short answer (SA) type questions, carrying* ***3*** *marks each.*

(*vi*) *In* ***Section D,*** *Question numbers* ***32*** *to* ***35*** *are long answer (LA) type questions carrying* ***5*** *marks each.*

(*vii*) *In* ***Section E,*** *Question numbers* ***36*** *to* ***38*** *are* ***case-study based integrated*** *questions carrying* ***4*** *marks each. Internal choice is provided in* ***2*** *marks question in each case-study.*

(*viii*) *There is no overall choice. However, an internal choice has been provided in 2 questions in Section B, 2 questions in Section* ***C****, 2 questions in Section* ***D*** *and 3 questions of* ***2*** *marks in Section* ***E****.*

(*ix*) *Draw neat diagrams wherever required. Take* $\pi = \frac{22}{7}$ *wherever required, if not stated.*

(*x*) *Use of calculators is* ***NOT allowed.***

SECTION—A

This section consists of 20 questions of 1 mark each.

1. HCF of 92 and 152 is :

(*a*) 4 (*b*) 19

(*c*) 26 (*d*) 45

Ans. (*a*)

2. The ratio of LCM and HCF of least composite number and least prime number is :

(*a*) 1 : 2 (*b*) 2 : 1

(*c*) 1 : 1 (*d*) 1 : 3.

Ans. (*b*)

3. LCM of 12, 15 and 21 is :

(*a*) 420 (*b*) 510

(*c*) 360 (*d*) 620.

Ans. (*a*)

4. If the sum of the zeroes of the polynomial $5x^2 - 4kx + 2$ is 8, then the value of k is :

(*a*) 12 (*b*) – 10

(*c*) 10 (*d*) 20.

Ans. (*c*)

5. If α, β are zeroes of $p(x) = 2x^2 - x - 6$, then value of $\alpha^{-1} + \beta^{-1}$ is :

(*a*) $\frac{1}{6}$ (*b*) $-\frac{1}{6}$

(*c*) $\frac{1}{2}$ (*d*) $-\frac{1}{3}$

Ans. (*b*)

6. The roots of the quadratic equation $x^2 + 2x - 35 = 0$ are :

(*a*) 5, 7 (*b*) – 5, 7

(*c*) – 5, – 7 (*d*) 5, – 7.

Ans. (*d*)

7. The next (4th) term of the A.P. $\sqrt{18}$, $\sqrt{50}$, $\sqrt{98}$, ... is :

(*a*) $\sqrt{128}$ (*b*) $\sqrt{140}$

(*c*) $\sqrt{162}$ (*d*) $\sqrt{200}$.

Ans. (*c*)

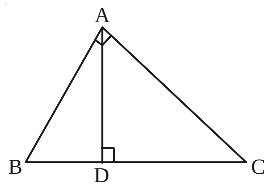
8. If the sum of p terms of an A.P. is $ap^2 + bp$, then its common difference is :

(*a*) $2a$ (*b*) $3a$

(*c*) a (*d*) $4a$.

Ans. (*a*)

9. In the given figure, if ∠BAC = 90° and AD ⊥ BC, then :

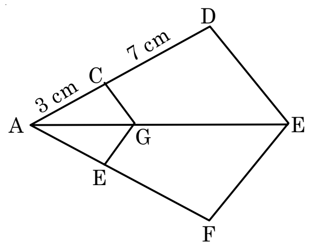


(*a*) $BD.CD = BC^2$ (*b*) $AB.AC = BC^2$

(*c*) $BD.CD = AD^2$ (*d*) $AB.AC = AD^2$.

Ans. (*c*)

10. In figure, GC || BD and GE || BF. If AC = 3 cm and CD = 7 cm, then value of $\frac{AE}{AF}$ is :



(*a*) $\frac{3}{10}$ (*b*) $\frac{7}{3}$

(*c*) $\frac{3}{7}$ (*d*) $\frac{10}{3}$.

Ans. (*a*)

11. If $\sin\theta = \frac{5}{13}$, then value of $\tan\theta$ is :

(*a*) $\frac{5}{12}$ (*b*) $\frac{11}{12}$

(*c*) $\frac{5}{4}$ (*d*) $\frac{1}{2}$.

Ans. (*a*)

12. If $\sec\theta = 2x$ and $\tan\theta = \frac{2}{x}$, then value of $\left(x^2 - \frac{1}{x^2}\right)$ is :

(*a*) 4 (*b*) $\frac{1}{4}$

(*c*) $\frac{1}{3}$ (*d*) 2.

Ans. (*b*)

13. A chord of a circle of radius 10 cm subtends a right angle at its centre. The length of the chord (in cm) is :

(*a*) $5\sqrt{2}$ (*b*) $10\sqrt{2}$

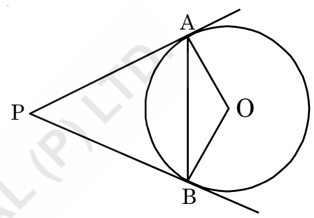
(*c*) $\frac{5}{\sqrt{2}}$ (*d*) 5.

Ans. (*b*)

14. In fig., PA and PB are tangents to the circle with centre O. If ∠APB = 60°, then ∠OAB is :

(*a*) 30° (*b*) 60°

(*c*) 90° (*d*) 15°



Ans. (*a*)

15. If the radius of the circle is diminished by 10%, then its area is diminished by :

(*a*) 12 % (*b*) 19 %

(*c*) 18 % (*d*) 25 %.

Ans. (*b*)

16. If the area of a circle is 616 cm^2, then its circumference is :

(*a*) 20 cm (*b*) 30 cm

(*c*) 44 cm (*d*) 88 cm.

Ans. (*d*)

17. The middle most obervation of every data arranged in order is called :

(*a*) Mode (*b*) Median

(*c*) Mean (*d*) Deviation.

Ans. (*b*)

18. Two dice are rolled simultaneously. What is the probability that 6 will come up at least once ?

(*a*) $\frac{1}{6}$ (*b*) $\frac{7}{36}$

(*c*) $\frac{11}{36}$ (*d*) $\frac{13}{36}$.

Ans. (*c*)

Direction: In the following questions, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**. Student studies the two statements labelled as Assertion (A) and Reason (R) and pick the correct option out of given four.

(*a*) Both Assertion (A) and Reason (R) are true and

Reason (R) is the correct explanation of the Assertion (A).

(*b*) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of the Assertion (A).

(*c*) Assertion (A) is true but Reason (R) is false.

(*d*) Assertion (A) is false but Reason (R) is true.

19. Assertion (A) : There exists only one real zeroes of $p(x) = (x - 2)(x^2 + 3)$.

Reason (R) : Sum of zeroes of a quadratic polynomial $ax^2 + bx + c$ is $-\frac{b}{a}$.

Sol. (*b*); $p(x) = 0$

$\Rightarrow (x - 2)(x^2 + 3) = 0$

$\Rightarrow x = 2, x^2 = -3$

$x = 2, x = \pm\sqrt{-3}$ (rejected as $\sqrt{-3}$ is non-real)

So, $x = 2$ means $p(x)$ has only one real zero.

(A) is true

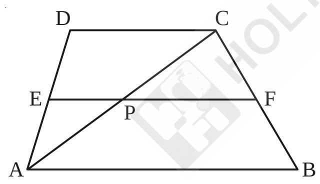
(R) is also true but it doesn't explain (A).

20. Assertion (A) : ABCD is a trapezium with DC || AB. E and F are points on AD and BC respectively, such that EF || AB. Then $\frac{AE}{ED} = \frac{BF}{FC}$.

Reason (R) : Any line parallel to parallel sides of a trapezium divides the non-parallel sides proportionally.

Sol. (*a*) In ΔABC, PF || AB

$$\therefore \quad \frac{AP}{PC} = \frac{BF}{FC} \quad ....(i)$$



In ΔACD, EP || DC

$$\therefore \quad \frac{AP}{PC} = \frac{AE}{ED} \quad ....(ii)$$

From equations (i) and (ii), we get

$$\frac{AE}{ED} = \frac{BF}{FC}$$

Hence Assertion (A) is true. Also Reason (R) is true and correct explanation of Assertion (A).

SECTION—B

This section consists of 5 questions of 2 marks each.

21. Find a quadratic polynomial whose one zero is $2 + \sqrt{5}$.

Sol. One zero = $2 + \sqrt{5}$ is an irrational.

We know that irrational zeroes occur as conjugate pairs.

$\therefore$ Other zero = $2 - \sqrt{5}$

$\therefore$ Sum of zeroes, S = $(2 + \sqrt{5}) + (2 - \sqrt{5}) = 4$

Product of zeroes, P = $(2 + \sqrt{5}) + (2 - \sqrt{5})$

$= (2)^2 - (\sqrt{5})^2 = 4 - 5 = -1$

$\therefore$ Quadratic polynomial,

$p(x) = x^2 - 5x + P = x^2 - 4x - 1.$

22. Points A (3, 1), B (5, 1), C (*a*, *b*) and D (4, 3) are vertices of a parallelogram ABCD. Find the values of *a* and *b*.

Sol. We know that diagonals of a parallelogram bisect each other.

$\therefore$ Mid point of diagonal AC = Mid point of diagonal BD

$$\left[\frac{a+3}{2}, \frac{b+1}{2}\right] = \left[\frac{4+5}{2}, \frac{3+1}{2}\right]$$

$\Rightarrow \quad \frac{a+3}{2} = \frac{4+5}{2}$ and $\frac{b+1}{2} = \frac{3+1}{2}$

$\Rightarrow \quad a + 3 = 9$ and $b + 1 = 4$

$\Rightarrow \quad a = 6$ and $b = 3$.

23. If $\cot\theta = \frac{4}{3}$, evaluate $\frac{4\sin\theta + 3\cos\theta}{4\sin\theta - 3\cos\theta}$.

Or

If tan A + cot A = 2, then find the value of $\tan^2 A + \cot^2 A$.

Sol. Given expression, $\frac{4\sin\theta + 3\cos\theta}{4\sin\theta - 3\cos\theta}$

Dividing numerator and denominator by sin θ.

$$= \frac{\frac{4\sin\theta}{\sin\theta} + \frac{3\cos\theta}{\sin\theta}}{\frac{4\sin\theta}{\sin\theta} - \frac{3\cos\theta}{\sin\theta}} = \frac{4 + 3\cot\theta}{4 - 3\cot\theta}$$

$$= \frac{4 + 3\left(\frac{4}{3}\right)}{4 - 3\left(\frac{4}{3}\right)} = \frac{4+4}{4-4} = \frac{8}{0} = \infty$$

(which is undefined)

Or

Sol. Given : tan A + cot A = 2

Squaring both sides, we get $(\tan A + \cot A)^2 = (2)^2$

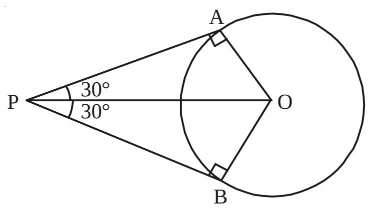
$\Rightarrow \tan^2 A + \cot^2 A + 2 \tan A \cot A = 4.$

$\Rightarrow \tan^2 A + \cot^2 A + 2 \times 1 = 4$

$\Rightarrow \tan^2 A + \cot^2 A = 2$

24. If two tangents inclined at an angle of 60° are drawn to a circle of radius 3 cm, then find the length of each tangent.

Sol. Let PA and PB are two tangents to the circle with centre O.



We know that, radius is perpendicular to the tangent at the point of contact

$\therefore \angle OAP = 90°$

$\angle OPA = \frac{1}{2}\angle APB = 30°$

In ΔOAP,

$\frac{PA}{OA} = \cot 30°$

$\Rightarrow \frac{PA}{3} = \sqrt{3} \Rightarrow PA = 3\sqrt{3}$ cm

25. Two different dice are tossed together. Find the probability :

(*i*) of getting a doublet

(*ii*) of getting a sum 10, of the numbers on the two dice.

Or

A card is drawn at random from a well-shuffled pack of 52 playing cards. Find the probability that the drawn card is neither a king nor a queen.

Sol. Total number of possible outcomes $n(S) = 36$

(*i*) Let E_1 be event of getting doublet.

Doublet are (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)

Total number of doublets, $n(E_1) = 6$

$\therefore$ P(getting a doublet) $= \frac{n(E_1)}{n(S)} = \frac{6}{36} = \frac{1}{6}$

(*ii*) Let E_2 be event of getting sum 10 on two dice.

Favourable outcomes for getting sum 10 are (4, 6), (5, 5), (6, 4) *i.e.*, $n(E_2) = 3$

$\therefore$ P (getting a sum 10) $= \frac{n(E_2)}{n(S)} = \frac{3}{36} = \frac{1}{12}$

Or

Sol. Let E be the event that the drawn card is neither a king nor a queen.

Total number of possible outcomes $n(S) = 52$

Total number of kings and queens = 4 + 4 = 8

Therefore, there are 52 – 8 = 44 cards that are neither king nor queen. *i.e.*, $n(E) = 44$

Total number of favourable outcomes = 44

$\therefore$ Required probability = P(E)

$= \frac{\text{Favourable outcomes}}{\text{Total number of outcomes}} = \frac{44}{52} = \frac{11}{13}$

SECTION—C

This section consists of 6 questions of 3 marks each.

26. Given that $\sqrt{3}$ is irrational, prove that $5 + 2\sqrt{3}$ is irrational.

Sol. Let us assume $5 + 2\sqrt{3}$ is rational, then it must be in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$

i.e. $5 + 2\sqrt{3} = \frac{p}{q}$

So, $\sqrt{3} = \frac{p - 5q}{2q}$ ....(*i*)

Since p, q, 5 and 2 are integers and $q \neq 0$. RHS of equation (*i*) is rational.

But LHS of (*i*) is $\sqrt{3}$ which is irrational. This is not possible.

This contradiction has arisen due to our wrong assumption that $5 + 2\sqrt{3}$ is rational. So $5 + 2\sqrt{3}$ is irrational.

27. For an A.P., it is given that the first term (a) = 5, common difference (d) = 3, and the nth term (a_n) = 50. Find n and sum of first n terms (S_n) of the A.P.

Or

Find the sum of all multiples of 7 lying between 500 and 900.

Sol. Here, $a = 5,\ d = 3,\ a_n = 50$

$\because \quad a_n = a + (n-1)d$

$\Rightarrow \quad 5 + (n-1)3 = 50$

$\Rightarrow \quad 3n + 2 = 50$

$\Rightarrow \quad 3n = 48$

$\Rightarrow \quad n = 16$

$$\because \quad S_n = \frac{n}{2}[2a + (n-1)\,d]$$

$$\therefore \quad S_{16} = \frac{16}{2}[2\,(5) + (16-1)\,3]$$

$$= 8\,[10 + 45] = 8\,(55) = 440.$$

Or

Sol. First multiple of 7 between 500 and 900 = 504

Last multiple of 7 between 500 and 900 =896

∴Required multiples of 7 are 504, 511, 518, ... , 896

Here, $a = 504,\ d = 7$

Let $a_n = l = 896$

Now, $a + (n-1)\,d = 896$

$\Rightarrow 504 + (n-1)\,7 = 896$

$\Rightarrow \quad 7n + 497 = 896$

$\Rightarrow \quad 7n = 399$

$\Rightarrow \quad n = 57$

$$\therefore \text{ Required sum } S_{57} = \frac{57}{2}[a + l]$$

$$= \frac{57}{2}[504 + 896] = \frac{57}{\cancel{2}} \times \overset{700}{\cancel{1400}} = 39900$$

Thus, the sum of all multiples of 7 lying between 500 and 900 is 39900.

28. Prove that: $\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1} = \operatorname{cosec} A + \cot A.$

Sol. L.H.S. $= \dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Multiply and divide by sin A.

$$= \frac{\sin A\,(\cos A - \sin A + 1)}{\sin A\,(\cos A + \sin A - 1)}$$

$$= \frac{\sin A \cos A - \sin^2 A + \sin A}{\sin A\,(\cos A + \sin A - 1)}$$

$$= \frac{\sin A \cos A + \sin A - (1 - \cos^2 A)}{\sin A\,(\cos A + \sin A - 1)}$$

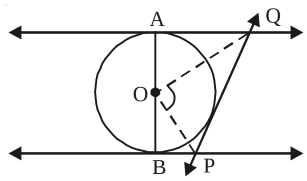
$$= \frac{\sin A\,(\cos A + 1) - (1 - \cos A)\,(1 + \cos A)}{\sin A\,(\cos A + \sin A - 1)}$$

$$= \frac{(1 + \cos A)\,(\sin A - 1 + \cos A)}{\sin A\,(\cos A + \sin A - 1)}$$

$$= \frac{1}{\sin A} + \frac{\cos A}{\sin A} = \operatorname{cosec} A + \cot A$$

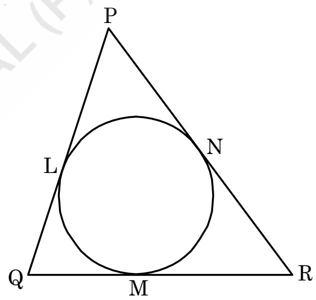
= R.H.S.

29. In the given figure, AB is a diameter of the circle with centre O. AQ, BP and PQ are tangents to the circle. Prove that ∠POQ = 90°.

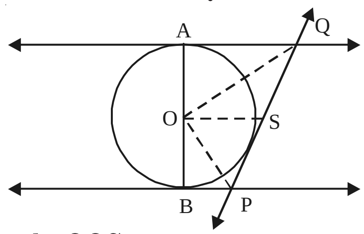


Or

In fig., a circle is inscribed in a ΔPQR with PQ = 10 cm, QR = 8 cm and PR = 12 cm. Find lengths QM, RN and PL.



Sol. The point of contact of PQ is S. Join OS.



In ΔQOA and ΔQOS

OQ = OA (common side)

OA = OS (radii of circle)

QA = QS (tangents drawn from an external point to a circle area equal]

∴ ΔQOA ≅ ΔQOS (by SSS congruency)

So, ∠QOA = ∠QOS (by cpet)

Similarly ΔPOB ≅ ∠POS

We know that ∠QOA + ∠QOS + ∠POB + ∠POS = 180°

∠QOS + ∠QOS + ∠POS + ∠POS = 180°

2 (∠QOS + ∠POS) = 180°

∠POQ = 90°.

Or

Sol. Since the length of tangents drawn from an external point are equal.

$\therefore$ QM = QL ... (1)

RM = RN ...(2)

PL = PN ... (3)

Now, PQ + QR + PR = 10 + 8 + 12

$\Rightarrow$ (PL + LQ) + (QM + MR) + (PN + NR) = 30

$\Rightarrow$ (PL + PN) + (LQ + QM) + (MR + NR) = 30

$\Rightarrow$ 2PL + 2QM + 2RN = 30 (Using (*1*), (*2*) (*3*))

$\Rightarrow$ PL + QM + RN = 15 ... (4)

Now, PQ = PL + LQ

$\Rightarrow$ 10 = PL + QM (Using (1))

$\Rightarrow$ PL + QM = 10 ... (5)

Subtracting (5) from (4),

RN = 15 – 10 = 5 cm ... (6)

Also, QR = QM + RN (Using (2))

$\Rightarrow$ QM + RN = 8 ...(7)

Subtracting (7) from (4),

PL = 15 – 8 = 7 cm ...(8)

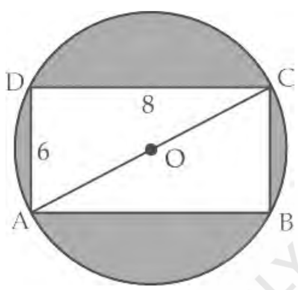
From (4), (6) and (8),

7 + QM + 5 = 15

$\Rightarrow$ QM = 3 cm

Thus, QM = 3 cm, RN = 5 cm and PL = 7 cm

30. Find the area of the shaded region in fig., if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take π = 3.14)



Sol. ΔABC is a right-angled triangle

$\therefore$ By Phythagoras, Theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow$ $AC = \sqrt{64+36} = 10$ cm

$\therefore$ Radius of the circle $(r) = \frac{AC}{2} = 5$ cm

Area of shaded region

= Area of circle – ar(ABCD)

$= \pi r^2 - l \times b = 3.14 \times (5)^2 - 6 \times 8$

$= 3.14 \times 25 - 6 \times 8 = 78.5 - 48 = 30.5$ cm^2

31. The probability of selecting a red ball at random from a jar that contains only red, blue and orange balls is $\frac{1}{4}$. The probability of selecting a blue ball at random from the same jar is $\frac{1}{3}$. If the jar contains 10 orange balls, find the total number of balls in the jar.

Sol. Here the jar contains red, blue and orange balls.

Let the number of red balls be x.

Let the number of blue balls be y.

Number of orange balls = 10.

Total number of balls = $x + y + 10$

$$P(\text{a red ball}) = \frac{x}{x+y+10}$$

$$\Rightarrow \frac{1}{4} = \frac{x}{x+y+10}$$

$\Rightarrow 4x = x + y + 10$

$\Rightarrow 3x - y = 10$ .....(1)

Next,

$$P(\text{a blue ball}) = \frac{y}{x+y+10}$$

$$\Rightarrow \frac{1}{3} = \frac{y}{x+y+10}$$

$\Rightarrow 3y = x + y + 10$

$\Rightarrow 2y - x = 10$ ....(2)

Eq. (1) × 2 + (2) $\Rightarrow$

$6x - 2y = 20$

$-x + 2y = 10$

$5x = 30$

$\Rightarrow x = 6$

Put $x = 6$ in Eq. (1), we get $y = 8$

Total number of balls = $x + y + 10 = 6 + 8 + 10 = 24$

Hence, total number of balls in the jar is 24.

SECTION—D

This section consists of 4 questions of 5 marks each.

32. The numerator of a fraction is 3 less than its denominator. If 2 is added to both the numerator and the denominator, then the sum of new fraction and original fraction is $\frac{29}{20}$. Find the original fraction.

Or

A train travels 360 km at a uniform speed. If the speed had been 5 km/hr more, it would

have taken 1 hr less for the same journey. Find the speed of the train.

Sol. Let the original fraction be $\frac{x}{(x+3)}$

According to the problem,

$$\Rightarrow \frac{x+2}{x+3+2}+\frac{x}{x+3}=\frac{29}{20}$$

$$\Rightarrow \frac{x+2}{x+5}+\frac{x}{x+3}=\frac{29}{20}$$

$$\Rightarrow \frac{(x+2)(x+3)+x(x+5)}{(x+5)(x+3)}=\frac{29}{20}$$

$$\Rightarrow \frac{(x^2+5x+6)+(x^2+5x)}{x^2+8x+15}=\frac{29}{20}$$

$$\Rightarrow 20(2x^2+10x+6)=29(x^2+8x+15)$$

$$\Rightarrow (40-29)x^2+(200-232)x+(120-435)=0$$

$$\Rightarrow 11x^2-32x-315=0.$$

$$\Rightarrow x=\frac{-(-32)\pm\sqrt{(-32)^2-4(11)(-315)}}{2\times 11}$$

$$=\frac{32\pm\sqrt{1024+13860}}{22}=\frac{32\pm 122}{22}$$

$$=\frac{154}{22},\frac{-90}{22}=7,\frac{-45}{11}$$

Rejecting $\frac{-45}{11}$

$\therefore$ Original fraction is $\frac{7}{10}$

Or

Sol. Let the original speed of the train be x km/hr.

Time taken to cover 360 km (t_1) $=\frac{360}{x}$ hrs.

Time taken to cover 360 km with increased speed of $(x+5)$ km/hr (t_2) $=\frac{360}{x+5}$ hours

$$\left[\because \text{Time}=\frac{\text{Distance}}{\text{Speed}}\right]$$

According to the problem, $t_1 - t_2 = 1$

$$\Rightarrow \frac{360}{x}-\frac{360}{x+5}=1$$

$$\Rightarrow \frac{360(x+5)-360(x)}{x(x+5)}=1$$

$$\Rightarrow \frac{360x+1800-360x}{x(x+5)}=1$$

$$\Rightarrow \frac{1800}{x^2+5x}=1$$

$$\Rightarrow 1800=x^2+5x$$

$$\Rightarrow x^2+5x-1800=0$$

$$\Rightarrow x^2+45x-40x-1800=0$$

$$\Rightarrow x(x+45)-40(x+45)=0$$

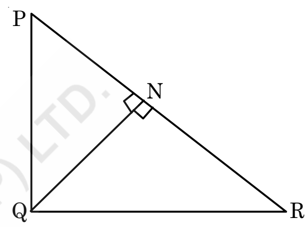
$$\Rightarrow (x+45)(x-40)=0$$

$$\Rightarrow x=-45 \text{ or } 40$$

But the speed can't be negative.

Hence, speed of the train is 40 km/hr.

33. In a ΔPQR, N is a point on PR, such that QN ⊥ PR. If PN × NR = QN², prove that ∠PQR = 90°.



Sol. **Given :** In ΔPQR, QN ⊥ PR

and PN.NR = QN^2

To prove : ∠PQR = 90°

Proof PN.NR = QN^2 [Given]

$$\therefore \frac{PN}{QN}=\frac{QN}{NR} \quad ....(i)$$

In ΔPNQ and ΔRNQ,

$$\frac{PN}{QN}=\frac{QN}{RN} \quad \text{(from } (i))$$

and ∠PNQ = ∠RNQ = 90° (∴ QN ⊥ PR)

∴ By SAS, ΔPNQ ~ ΔQNR

⇒ ∠P = ∠NQR ....(*ii*)

⇒ ∠R = ∠NQP ....(*iii*)

In ΔPQR,

∠P + ∠NQP + ∠NQR + ∠R = 180° (ASP)

⇒ ∠NQR + ∠NQP + ∠NQR + ∠NQP = 180° *[using (ii) and (iii)]*

⇒ 2 (∠NQP + ∠NQR) = 180°

$$\Rightarrow \angle NQP + \angle NQR = \frac{180°}{2}$$

⇒ ∠Q = 90°

⇒ ∠PQR = 90°.

[Hence proved]

34. A statue 1.6 m tall, stands on the top of a pedestal. From a point on the ground, the angle of elevation of the top of the statue is 60° and from the same point the angle of elevation of the top of the pedestal is 45°. Find the height of the pedestal.

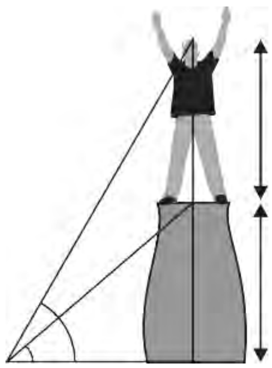
(Use $\sqrt{3}$ = 1.73)

OR

The angle of elevation of the top Q of a vertical tower PQ from a point X on the ground is 60°. From a point Y, 40 m vertically above X, the angle of elevation of the top Q of tower is 45°. Find the height of the tower PQ and the distance PX. (Use $\sqrt{3}$ = 1.73)

Sol. Let AB be the pedestal and AC be the statue. Given, AC = 1.6 m

Angle of elevation of the top of the statue = ∠CPB = 60°



Angle of elevation of the top of the pedestal = ∠APB = 45°

Let the height of the pedestal be h m.

In rt. ∠d ΔABP,

$$\tan 45° = \frac{AB}{PB} \Rightarrow 1 = \frac{h}{BP}$$

$$h = BP \quad ....(i)$$

In rt. ∠d ΔPBC,

$$\tan 60° = \frac{BC}{BP} \Rightarrow \sqrt{3} = \frac{h+1.6}{BP}$$

$$\Rightarrow \quad BP = \frac{(h+1.6)}{\sqrt{3}} \quad ...(ii)$$

$$(i) \text{ and } (ii) \Rightarrow \quad h = \frac{h+1.6}{\sqrt{3}}$$

$$\Rightarrow \quad h\sqrt{3} = h + 1.6$$

$$\Rightarrow \quad h(\sqrt{3}-1) = 1.6$$

$$\Rightarrow \quad h(1.73-1) = 1.6$$

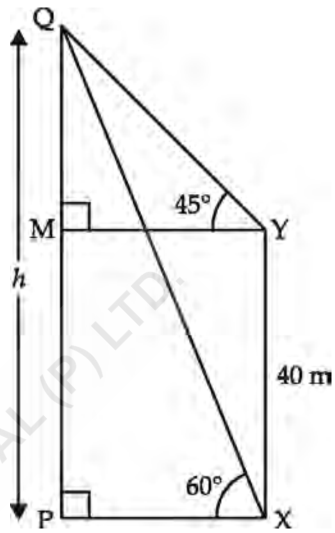
$$\Rightarrow \quad h(0.73) = 1.6$$

$$\Rightarrow \quad h = \frac{1.6}{0.73} = 2.19 \text{ m}$$

∴ Height of the pedestal is 2.19 m.

Or

Sol.



Let PQ = h

MP = YX = 40 m

∴ QM = $h - 40$

In right-angled ΔQMY,

$$\tan 45° = \frac{QM}{MY} \Rightarrow 1 = \frac{h-40}{PX} \quad (\because MY = PX)$$

$$\therefore \quad PX = h - 40 \quad ....(i)$$

In right-angled ΔQPX,

$$\tan 60° = \frac{QP}{PX} \Rightarrow \sqrt{3} = \frac{h}{PX}$$

$$\Rightarrow \quad PX = \frac{h}{\sqrt{3}} \quad ...(ii)$$

From (*i*) and (*ii*)

$$h - 40 = \frac{h}{\sqrt{3}}$$

$$\Rightarrow \quad \sqrt{3}(h-40) = h$$

$$\Rightarrow \quad \sqrt{3}h - h = 40\sqrt{3}$$

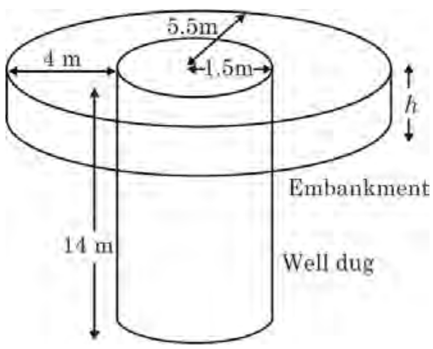
$$\Rightarrow \quad 1.73h - h = 40(1.73)$$

$$\Rightarrow \quad h(1.73-1) = 40(1.73)$$

$$\Rightarrow \quad h = 94.79 \text{ m}$$

35. A well of diameter 3 m is dug 14 m deep. The earth taken out of it has been spread evenly all around it in the shape of a circular ring of width 4 m to form an embankment. Find the height of the embankment.

Sol. Here, depth of well (H) = 14 m



Radius of well (r) = $\frac{3}{2} = 1.5$ m

Volume of earth dug out = $\pi r^2 H$

$= \pi (1.5)^2 \times 14 = \frac{63}{2}$ p m^2 ....(1)

Outer radius of embankment R = 4 + 1.5 = 5.5 m

Area of base of embankment = $\pi (R^2 - r^2)$

$= \pi [(5.5)^2 - (1.5)^2]$

$= \pi (5.5 + 1.5)(5.5 - 1.5)$

$= 28 \pi m^2$

Let h be height of embankment.

$\therefore$ Volume of embankment

= (Area of base) × Height = $(28\pi) h$ ....(2)

But volume of each dug out = Volume of embankment

$\Rightarrow \frac{63\pi}{2} = 28 \pi h$

$\Rightarrow h = \frac{63}{28 \times 2} = \frac{9}{8} = 1.125$ m

SECTION—E

This section consists of 3 Case-Study Based Questions of 4 marks each.

36. Suryansh writes test to upgrade her level. The test has 25 questions for a total score of 150 points. Among the 25 questions, each multiple choice question carries 3 marks and the descriptive type questions carries 8 marks.

Study the following situation and answer the following.

(*i*) If number of MCQ's be x and descriptive type questions be y, then represent the given situation.

(*ii*) Find the number of MCQ asked in test.

(*iii*) Find number of descriptive type questions asked in the test.

OR

How much marks Suryansh score if he has attempted all MCQ's correctly and 5 descriptive type questions incorrectly ?

Sol. (*i*) $x + y = 25$ and $3x + 8y = 150$

(*ii*) $8(x + y) - (3x + 8y) = 8 \times 25 - 150$

$\Rightarrow 8x + 8y - 3x - 8y = 200 - 150$

$\Rightarrow 5x = 50 \Rightarrow x = 10$

(*iii*) $3(x + y) - (3x + 8y) = 3 \times 25 - 150$

$\Rightarrow 3x + 3y - 3x - 8y = 75 - 150 \Rightarrow -5y = -75 \Rightarrow y = 15$

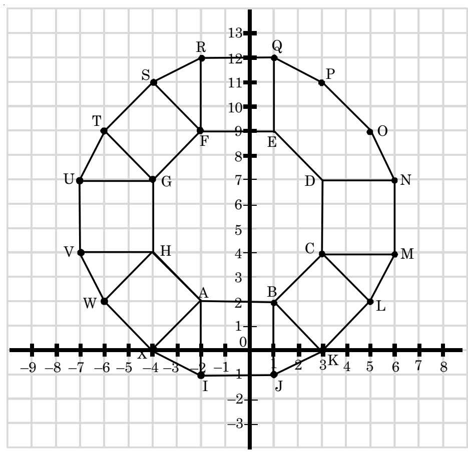
Or

$3x + 8(y - 5) = 3 \times 10 + 8(15 - 5)$

$= 30 + 80 = 110$

37. A tiling or tessellation of a flat surface is the covering of a plane using one or more geometric shapes, called tiles, with no overlaps and no gaps. Historically, tessellations were used in ancient Rome and in Islamic art. You may find tessellation patterns on floors, walls, paintings, etc. Shown below is a tiled floor in the archaeological museum of Seville, made using squares, triangles and hexagons.

A craftsman thought of making a floor pattern after being inspired by the above design. To ensure accuracy in his work, he made the pattern on the Cartesian plane. He used regular octagons, squares and triangles for his floor tessellation pattern.



Use the above figure to answer the questions that follow :

(*i*) What is the length of the line segment joining points B and F ?

(*ii*) The centre 'Z' of the figure will be the point of intersection of the diagonals of quadrilateral WXOP. Then what are the coordinates of Z ?

(*iii*) What are the coordinates of the point on *y*-axis equidistant from A and G ?

OR

What is the area of Trapezium AFGH ?

Sol. **(*i*)** B (1, 2), F (– 2, 9)

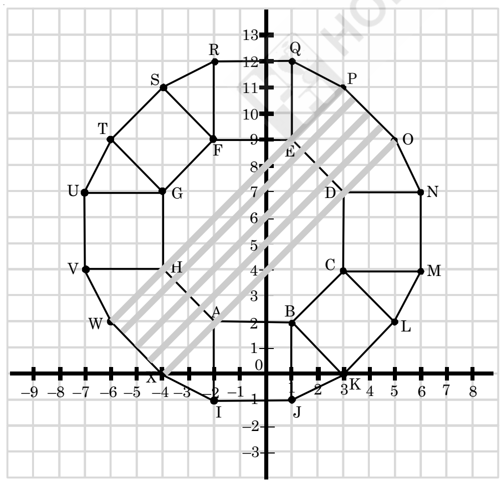
$BF^2 = (-2-1)^2 + (9-2)^2$

$= (-3)^2 + (7)^2$

$= 9 + 49 = 58$

So, BF = $\sqrt{58}$ units

(*ii*)



W (– 6, 2), X (– 4, 0), O (5, 9), P (3, 11)

Clearly WXOP is a rectangle

Point of intersection of diagonals of a rectangle is the mid-point of the diagonals. So, the required point is mid point of WO or XP.

$$= \left(\frac{-6+5}{2}, \frac{2+9}{2}\right) = \left(\frac{-1}{2}, \frac{11}{2}\right).$$

(*iii*) A (– 2, 2), G (– 4, 7)

Let the point on *y*-axis be Z (0, *y*)

$$AZ^2 = GZ^2$$

$$(0+2)^2 + (y-2)^2 = (0+4)^2 + (y-7)^2$$

$$(2)^2 + y^2 + 4 - 4y = (4)^2 + y^2 + 49 - 14y$$

$$8 - 4y = 65 - 14y$$

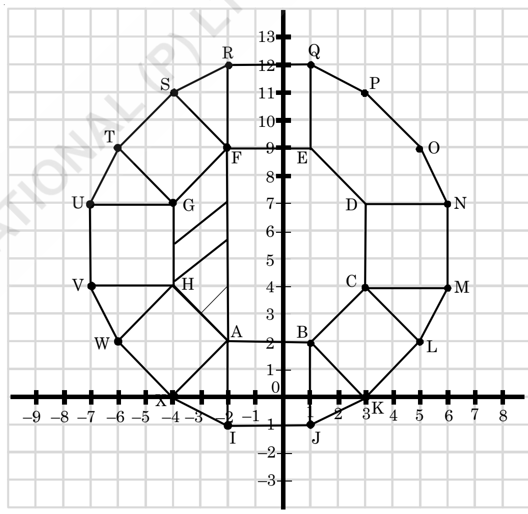
$$10y = 57$$

So, $y = 5, 7$

i.e., the required point is (0, 5.7).

Or

Sol.



A (– 2, 2), F (– 2, 9), G (– 4, 7), H (– 4, 4)

Clearly, GH = $\sqrt{(-4+4)^2 + (7-4)^2} = \sqrt{9}$

= 3 units

AF = $\sqrt{(-2+2)^2 + (2-9)^2} = \sqrt{49}$

= 7 units

So, height of the trapezium APGH = 2 units

So, area of AFGH = $\frac{1}{2}(AF + GH) \times$ height

$= \frac{1}{2}(7+3) \times 2$

= 10 sq. units.

38. IPL : Indian premier league is a professional T-20 cricket league in India contested usually during summer when most international cricket is not happening. Team 'X' is there since the beginning in 2008.

Its record of first seven seasons of IPL is given in tabular form.

Season	Matches won	Matches lost	Total matches played (*c.f.*)
1	5	3	8
2	4	4	16
3	5	2	23
4	6	3	32
5	2	4	38
6	4	2	44
7	3	3	50

Answer the following :

(*i*) What % of the game did Team 'X' win ?

(*ii*) What is median of matches lost ?

Or

Team X ended first 13 seasons with 70% winning percentage. How many matches must they have won out of 50 matches that were played from season 8 to season 13 ?

(*iii*) Which season was the worst for team 'X' ?

Sol. (*i*) Team X won 29 games out of 50 games.

($\therefore$ N = 50)

$$\therefore \text{Winning } \% = \frac{29}{50} \times 100 = 58\%$$

(*ii*) Listing matches lost in descending order as 4, 4, 3, 3, 3, 2, 2.

Here N = 7 (odd)

$$\therefore \text{ Median} = \left(\frac{7+1}{2}\right)^{th} \text{ item} = 4^{th} = \text{item} = 3$$

Or

To win 70% of all games played till 13^{th} season, the team must win 70 out of total 100 games.

$\Rightarrow$ Team must win 70 – 29 = 41 games out of remaining 50 games.

(*iii*) Since number of wins – No. of losses is the least for 5th season (2 – 4 = – 2 *i.e.*, total of 2 losses)

$\therefore$ Fifth week is worst.

Holy Faith New Style Sample Paper–4

(Based on the Latest Design & Syllabus Issued by CBSE)

CLASS — 10th (CBSE)
MATHEMATICS (STANDARD)

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instructions : Same as in Holy Faith New Style Sample Paper—1.

SECTION—A

This section consists of 20 questions of 1 mark each.

1. If LCM (x, 18) = 36 and HCF (x, 18) = 2, then x is :

(a) 2 (b) 3
(c) 4 (d) 5

Ans. (d)

2. If two positive integers p and q can be expressed as $p = ab^2$ and $q = a^3b$; a, b being prime numbers, then LCM (p, q) is :

(a) a^2b^2 (b) a^3b^3
(c) ab (d) a^3b^2.

Ans. (d)

3. The product of non-zero rational and an irrational number is :

(a) always irrational (b) always rational
(c) rational or irrational (d) one.

Ans. (a)

4. If α and β are zeroes of $3x^2 + 15x - 12$, then the value of $\alpha\beta$ is :

(a) – 4 (b) 4
(c) 5 (d) – 5.

Ans. (a)

5. The graph of a polynomial P (x) cuts the x-axis at 3 points and touches it at 2 other points. The number of zeroes of P (x) is :

(a) 1 (b) 2
(c) 3 (d) 5.

Ans. (d)

6. The pair of equations $ax + 2y = 9$ and $3x + by = 18$ represent parallel lines, where a and b are integers, if :

(a) $a = b$ (b) $3a = 2b$
(c) $2a = 3b$ (d) $ab = 6$.

Ans. (d)

7. If 1 is root of the equations $ay^2 + ay + 3 = 0$ and $y^2 + y + b = 0$ then ab equals :

(a) 3 (b) $-\frac{7}{2}$
(c) 6 (d) – 3.

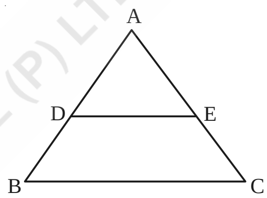
Ans. (a)

8. The number of two-digit numbers which are divisible by 3 is :

(a) 26 (b) 30
(c) 40 (d) 32.

Ans. (b)

9. In the given figure, in ΔABC, DE || BC. If AD = 2.4 cm, DB = 4 cm and AE = 2 cm, then the length of AC is :



(a) $\frac{10}{3}$ cm (b) $\frac{3}{10}$ cm
(c) $\frac{16}{3}$ cm (d) 1.2 cm.

Ans. (a)

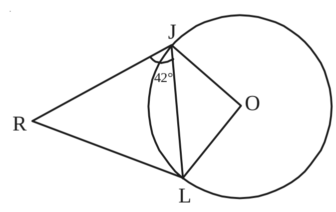
10. If the ratio of the perimeter of ΔAOB to the perimeter of ΔCOD would have been 1 : 4, then :

(a) AB = 2 CD (b) AB = 4 CD
(c) CD = 2 AB (d) CD = 4 AB.

Ans. (d)

11. In ΔABC right-angled at B, sin A = $\frac{7}{25}$, then the value of cos C is :

(a) $\frac{7}{25}$ (b) $\frac{24}{25}$
(c) $\frac{7}{24}$
(d) $\frac{24}{7}$.



Ans. (a)

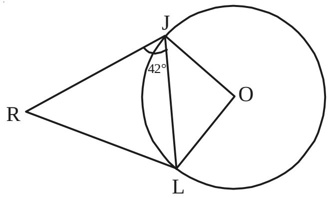
12. If $\sec^2\theta\,(1 + \sin\theta)(1 - \sin\theta) = k$, then the value of k is :

(a) 1 (b) – 1

(c) $\frac{1}{2}$ (d) not defined.

Ans. (a)

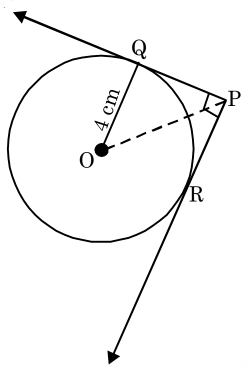
13. In the given figure, RJ and RL are two tangents to the circle. If $\angle$RJL = 42°, then the measure of $\angle$JOL is :



(a) 42° (b) 84°
(c) 96° (d) 138°.

Ans. (b)

14. In the figure, from an external point P, two tangents PQ and PR are drawn to a circle of radius 4 cm with centre O. If $\angle$QPR = 90°, then length of PQ is :



(a) 3 cm (b) 4 cm
(c) 2 cm (d) $2\sqrt{2}$ cm.

Ans. (b)

15. What is the area of a semi-circle of diameter 'd' ?

(a) $\frac{1}{16}\pi d^2$ (b) $\frac{1}{4}\pi d^2$
(c) $\frac{1}{8}\pi d^2$ (d) $\frac{1}{2}\pi d^2$.

Ans. (c)

16. A cylinder, a cone and a hemisphere are of equal base and have same height. The ratio of their volumes is :

(a) 1 : 3 : 2 (b) 3 : 2 : 1
(c) 3 : 1 : 2 (d) 5 : 1 : 3.

Ans. (c)

17. For same data, $x_1, x_2, \ldots\ldots, x_n$ with respective frequencies $f_1, f_2, \ldots\ldots, f_n$, the volume of $\sum_{1}^{n} f_i (x_i - \bar{x})$ is equal to :

(a) $n\bar{x}$ (b) 1
(c) Σf_i (d) 0.

Ans. (d)

18. The probability of getting an even number, when a die is thrown once is :

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$
(c) $\frac{1}{6}$ (d) $\frac{5}{6}$.

Ans. (a)

Direction: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option.

(a) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(b) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(c) Assertion (A) is true but reason (R) is false.

(d) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : Polynomials having – 3 and 5 as its zeroes are infinite in number

Reason (R) : A polynomial whose sum of zeroes and product of zeroes are – 5 and 6, respectively is $x^2 + 5x + 6$.

Sol. (b); (A) is true as polynomials having zeroes α and β are given by $k\{x^2 - (\alpha + \beta)x + \alpha\beta\}$, where k is any real number.

(R) is also true as by taking value of k as 1, one polynomial becomes

$1\{x^2 - (-5)x + 6\} = x^2 + 5x + 6$

But (R) doesn't explain (A).

20. Assertion (A) : ΔABC ~ ΔDEF such that $\angle$A = 45°, $\angle$E = 56° $\Rightarrow$ $\angle$C = 79°.

Reason (R) : Corresponding angles of similar triangles are equal.

Sol.(a); $\Delta ABC \sim \Delta DEF$

$\Rightarrow \angle B = \angle E = 56°$

In ΔABC,

$$\angle C = 180° - (\angle A + \angle B)$$
$$= 180° - (45° + 56°)$$
$$= 180° - 101°$$
$$= 79°$$

So, (A) is true. (R) is also true and it explains (A).

SECTION—B

This section consists of 5 questions of 2 marks each.

21. If one zero of the polynomial $ax^2 + bx + c$ is triple of the other, then show that $3b^2 = 16ac$.

Sol. Let two zeroes be α and 3α. Then

$$\alpha + 3\alpha = \frac{-b}{a} \Rightarrow 4\alpha = \frac{-b}{a} \Rightarrow \alpha = \frac{-b}{4a}$$

and $$\alpha(3\alpha) = \frac{c}{a} \Rightarrow 3\alpha^2 = \frac{c}{a}$$

$$\Rightarrow \quad 3\left(\frac{-b}{4a}\right)^2 = \frac{c}{a} \Rightarrow \frac{3b^2}{16a^2} = \frac{c}{a}$$

$$\Rightarrow \quad 3b^2 = 16ac.$$

22. Prove that the points (3, 0), (6, 4) and (– 1, 3) are the vertices of an isosceles triangle.

OR

Find the radius of the circle whose end points of diameter are (24, 1) and (2, 23).

Sol. Let the given points be A (3, 0), B (6, 4) and C (– 1, 3)

$$\therefore \quad AB = \sqrt{(6-3)^2 + (4-0)^2}$$

$$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$BC = \sqrt{(-1-6)^2 + (3-4)^2}$$

$$= \sqrt{(-7)^2 + (-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$$

$$CA = \sqrt{(3+1)^2 + (0-3)^2}$$

$$= \sqrt{(4)^2 + (-3)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$\therefore \quad AB = CA = 5$$

Hence, ΔABC is an isosceles triangle.

Or

Sol. Let A (24, 1) and B (2, 23) be end points of diameter AB.

$$\therefore \quad AB = \sqrt{(2-24)^2 + (23-1)^2}$$

$$= \sqrt{(-22)^2 + (22)^2} = 22\sqrt{2} \text{ units}$$

$$\therefore \text{ Radius of circle} = \frac{AB}{2} = \frac{22\sqrt{2}}{2} = 11\sqrt{2} \text{ units.}$$

23. If $a \cos\theta + b \sin\theta = m$ and $a \sin\theta - b \cos\theta = n$, then prove that $a^2 + b^2 = m^2 + n^2$.

Sol. Given : $a \cos\theta + b \sin\theta = m$ ...(*i*)

$a \sin\theta - b \cos\theta = n$ ...(*ii*)

Squaring and adding (*i*) + (*ii*) we get

$(a \cos\theta + b \sin\theta)^2 + (a \sin\theta - b \cos\theta)^2 = m^2 + n^2$

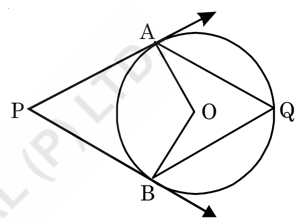
$\Rightarrow a^2 \cos^2\theta + b^2 \sin^2\theta + 2ab \sin\theta \cos\theta + a^2 \sin^2\theta + b^2 \cos^2\theta - 2ab \sin\theta \cos\theta = m^2 + n^2$

$\Rightarrow a^2 (\sin^2\theta + \cos^2\theta) + b^2 (\sin^2\theta + \cos^2\theta) = m^2 + n^2$

$\Rightarrow a^2 + b^2 = m^2 + n^2 (\because \sin^2\theta + \cos^2 = 1)$

Hence proved

24. In the given figure, O is the centre of circle. Find ∠AQB, given that PA and PB are tangents to the circle and ∠APB = 75°.



Sol. $\angle PAO = \angle PBO = 90°$

(angle between radius and tangent)

In quadrilateral AOBP,

$75° + 90° + 90° + \angle AOB = 360°$

$\Rightarrow \quad \angle AOB = 360° - (75° + 90° + 90°)$

$\Rightarrow \quad \angle AOB = 105°$

$$\Rightarrow \quad \angle AQB = \frac{1}{2} \times 105° = 52.5°$$

(Angle at the remaining part of the circle is half the angle subtended by the arc at the centre)

25. In a pack of 52 playing cards one card is lost. From the remaining cards, a card is drawn at random. Find the probability that the drawn card is queen of heart, if the lost card is a black card.

Or

A child has a die whose 6 faces show the letters given below :

A B C A A B

The die is thrown once. What is probability of getting (*i*) A (*ii*) B ?

Sol. Total cards = 52 – 1 = 51

Total outcomes = 51

Favourable outcome = 1 [queen of heart]

Let E denote the event of getting probability of queen of heart

$\therefore \quad P(E) = \frac{1}{51}.$

Or

Sol. Here, S = {A, B, C, A, A, B}

$\therefore$ Total no. of possible outcomes, n (S) = 6

(*i*) No. of outcomes favourable of getting A, n (getting A) = 3

$\therefore P(\text{getting A}) = \frac{n(\text{getting A})}{n(S)} = \frac{3}{6} = \frac{1}{2}$

(*ii*) No. of outcomes favourable of getting B, n (getting B) = 2

$\therefore P(\text{getting B}) = \frac{n(\text{getting B})}{n(S)} = \frac{2}{6} = \frac{1}{3}.$

SECTION—C

This section consists of 6 questions of 3 marks each.

26. In a teachers workshop, the number of teachers teaching French, Hindi and English are 48, 80 and 144 respectively. Find the minimum number of rooms required if in each room the same number of teachers are seated and all of them are of the same subject.

Sol.

$$48 = 2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3^1$$
$$80 = 2 \times 2 \times 2 \times 2 \times 5 = 2^4 \times 5^1$$
$$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$$

HCF of 48, 80 and 144 = $2^4 = 16$

$\therefore$ Number of teachers seated in a room = 16

Number of rooms required

$$= \frac{48 + 80 + 144}{16} = \frac{272}{16} = 17$$

27. An A.P. consists of 21 terms, The sum of the three terms on the middle is 129 and of the last three is 237. find the A.P.

Sol. Let a be the first term and d be the common difference of the given A.P.

It is given that $a_{10} + a_{11} + a_{12} = 129$ and $a_{19} + a_{20} + a_{21} = 237$

$\therefore (a + 9d) + (a + 10d) + (a + 11d) = 129$ and $(a + 18d) + (a + 19d) + (a + 20d) = 237$

$\Rightarrow \quad 3a + 30d = 129$ and $3a + 57d = 237$

Subtracting these equations, we get :

$3a + 57d - (3a + 30d) = 237 - 129$

$\Rightarrow \quad 57d - 30d = 108$

$\Rightarrow \quad 27d = 108 \Rightarrow d = 4$

Putting value of d in $3a + 30d = 129$, we get

$3a + 30(4) = 129$

$\Rightarrow \quad 3a = 129 - 120 \Rightarrow a = 3$

Thus, required A.P. is 3, 7, 11, 15, ... , 79, 83.

28. If 4 tan θ = 3, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$.

Or

Prove that : $\frac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)} = \cot\theta$..

Sol. Given : $4\tan\theta = 3 \Rightarrow \tan\theta = \frac{3}{4}$

Using trigonometric identities, we have

$$\sec\theta = \sqrt{1 + \tan^2\theta} = \sqrt{1 + \frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$\cos\theta = \frac{1}{\sec\theta} = \frac{4}{5}$$

$$\sin\theta = \sqrt{1 - \cos^2\theta} = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}} = \frac{3}{5}$$

$$\therefore \frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1} = \frac{4 \times \frac{3}{5} - \frac{4}{5} + 1}{4 \times \frac{3}{5} + \frac{4}{5} - 1} = \frac{13}{11}$$

Or

Sol. L.H.S. $= \frac{1 + \cos\theta - \sin^2\theta}{\sin\theta(1 + \cos\theta)}$

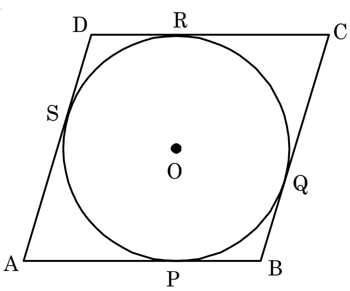
$$= \frac{(1 - \sin^2\theta) + \cos\theta}{\sin\theta(1 + \cos\theta)}$$

$$= \frac{\cos^2\theta + \cos\theta}{\sin\theta(1 + \cos\theta)} = \frac{\cos\theta(\cos\theta + 1)}{\sin\theta(1 + \cos\theta)}$$

$= \cot\theta$ = R.H.S. **Q.E.D.**

29. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. Given : ABCD is a parallelogram such that its sides touching a circle with centre at O at P, Q, R and S.



To prove : AB = BC = CD = DA

Proof :

∴ Tangents drawn from an external point are equal.

∴ AP = AS ... (1)

BP = BQ ... (2)

CR = CQ ... (3)

and DR = DS ... (4)

Adding (1), (2), (3) and (4), we get

AP + BP + CR + DR = AS + BQ + CQ + DS

⇒ (AP + BP) + (CR + DR) = (AS + DS) + (BQ + CQ)

⇒ AB + CD = AD + BC

⇒ 2 AB = 2 BC

(∵ ABCD is a parallelogram ∴ AB = CD and BC = AD)

⇒ AB = BC

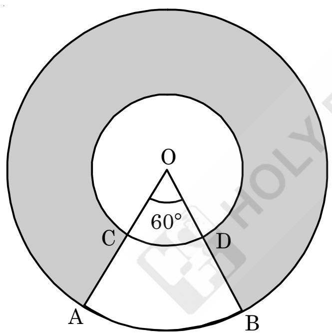
∴ AB = BC = CD = AD

Thus, ABCD is a rhombus.

Q.E.D.

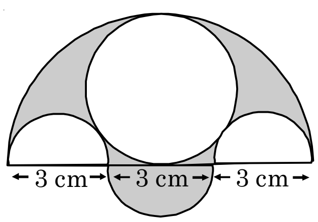
30. In fig., two concentric circles with centre O, have radii 21 cm and 42 cm; if ∠AOB = 60°, find the area of the shaded region.

$\left(\textbf{Use } \pi = \frac{22}{7}\right)$



OR

Three semicircles each of diameter 3 cm, a circle of diameter 4.5 cm and a semicircle of radius 4.5 cm are drawn in the given figure. Find the area of the shaded region.



Sol. Here, outer radii (R) = 42 cm,

Inner radii(r) = 21 cm

Area of the ring $= \pi (R^2 - r^2)$

$$= \frac{22}{7}\{(42)^2 - (21)^2\} = \frac{22}{7}(42 - 21)(42 + 21)$$

$$= \frac{22}{7} \times 21 \times 63 = 4158 \text{ cm}^2$$

Area of sector of inner circle of radius 21 cm. (A_1)

$$= \frac{\pi r^2 \times 60°}{360°} = \frac{22}{7} \times 21 \times 21 \times \frac{60°}{360°}$$

$= 11 \times 21 = 231 \text{ cm}^2$

Area of sector of outer circle of radius 42 cm (A_2)

$$= \frac{\pi R^2 \times 60°}{360°} = \frac{22}{7} \times 42 \times 42 \times \frac{1}{6} = 924 \text{ cm}^2$$

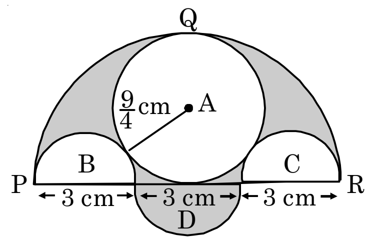
Area of shaded region

= Area of ring – $[A_2 - A_1]$ = 4158 – 924 + 231

= 3465 cm^2.

Or

Sol. Let us mark various points P, Q and R. Also, mark all three regions of smaller circle, three semicircles by B, C and D as shown in the figure and A in bigger circle.



Radius of biggest semicircle PQR (r) $= \frac{3+3+3}{2}$

$$= \frac{9}{2} \text{cm}$$

Area of semicircle PQR $= \frac{1}{2}\pi(r^2) = \frac{1}{2}\pi\left(\frac{9}{2}\right)^2$

$$= \frac{81}{8}\pi \text{ cm}^2$$

Radius of circle (A) $= \frac{4.5}{2} = \frac{9}{2 \times 2} \text{cm} = \frac{9}{4} \text{cm}$

Area of region A = $\pi\left(\frac{9}{4}\right)^2 = \frac{81\pi}{16}\text{cm}^2$

Area of region (B + C) = 2 × Area of semicircles

$$= 2\times\frac{1}{2}\pi\left(\frac{3}{2}\right)^2 = \frac{9\pi}{4}\text{cm}^2$$

Area of region D = $\frac{1}{2}\pi\left(\frac{3}{2}\right)^2 = \frac{9\pi}{8}\text{ cm}^2$

∴ Area of shaded region = Area of semicircle – Area of region A – (Area of region (B) + Area of region (C)} + Area of region D

$$= \frac{81\pi}{8} - \frac{81\pi}{16} - \frac{9\pi}{4} + \frac{9\pi}{8} = \frac{63\pi}{16}\text{cm}^2 = \frac{99}{8}\text{cm}^2.$$

31. Two coins are tossed simultaneously. What is the probability of getting:

(*i*) At least one head ?

(*ii*) At most one tail ?

(*iii*) A head and a tail ?

Sol. When two coins are tossed, the total outcomes are

{HH, HT, TH, TT}

(*i*) P (At least one head) = $\frac{3}{4}$

(*ii*) P (At most one tail) = $\frac{3}{4}$

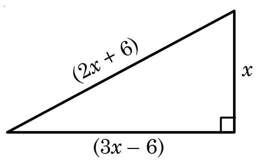
(*iii*) P (A head and a tail) = $\frac{2}{4} = \frac{1}{2}$.

SECTION—D

This section consists of 4 questions of 5 marks each.

32. The hypotenuse (in cm) of a right-angled triangle is 6 cm more than twice the length of the shortest side. If the length of the third side is 6 cm less than thrice the length of shortest side, then find the dimensions of the triangle.

Sol. Let shortest side of right angled triangle be *x*.



∴ Hypotenuse = $(2x + 6)$ cm

Second side = $(3x – 6)$ cm

Using Pythagoras Theorem,

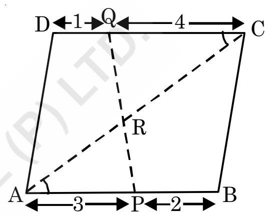
$$(2x + 6)^2 = x^2 + (3x – 6)^2$$
$$4x^2 + 36 + 24x = x^2 + 9x^2 + 36 – 36x$$
$$6x^2 – 60x = 0$$
$$6x\,(x – 10) = 0 \Rightarrow x = 0,\ 10$$

Rejecting $x = 0$, ⇒ $x = 10$ cm

∴ Remaining sides are (3 × 10 – 6) = 24 cm and (2 × 10 + 6) cm = 26 cm

∴ Dimensions of triangle are 10 cm, 24 cm and 26 cm.

33. ABCD is a || gm. AB is divided at P and CD at Q so that AP : PB = 3 : 2 and CQ : QD = 4 : 1. If PQ meets AC at R, then prove that AR = $\frac{3}{7}$AC.



Sol. Let AB = CD = x

$$AP = \frac{3}{5}AB = \frac{3}{5}x$$

$$CQ = \frac{4}{5}CD = \frac{4}{5}x$$

Now, in ΔARP and ΔCRQ

∠PAR = ∠RCQ (∵ AB || CD)

∠ARP = ∠CRQ (Vertically opp. angles)

⇒ ΔARP ~ ΔCRQ (By AA similarity)

$$\Rightarrow \frac{AR}{CR} = \frac{AP}{CQ}$$

$$\Rightarrow \frac{AR}{CR} = \frac{\frac{3}{5}x}{\frac{4}{5}x}$$

$$\Rightarrow \frac{AR}{CR} = \frac{3}{4}$$

$$\Rightarrow \frac{CR}{AR} = \frac{4}{3}$$

Adding 1 on both sides,

$$\Rightarrow \frac{CR}{AR}+1 = \frac{4}{3}+1$$

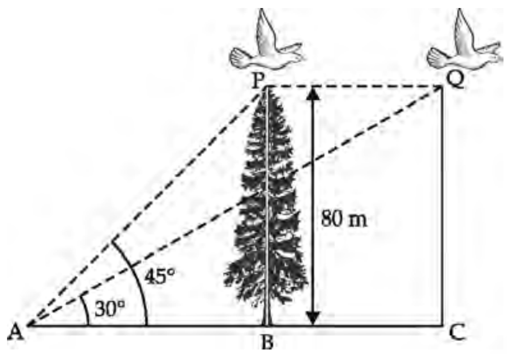
$$\Rightarrow \quad \frac{CR + AR}{AR} = \frac{4+3}{3}$$

$$\frac{AC}{AR} = \frac{7}{3}$$

$$\Rightarrow \quad AR = \frac{3}{7}AC \qquad \textbf{Q.E.D.}$$

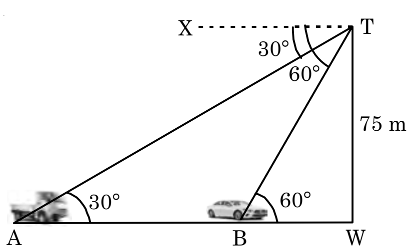
34. A bird is sitting on the top of 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird.

(Take $\sqrt{3}$ = 1.732)



Or

A straight highway leads to the foot of a tower. A man standing on the top of the 75 m high tower observes two cars at angles of depression of 30° and 60°, which are approaching the foot of the tower. If one car is exactly behind the other on the same side of the tower, find the distance between the two cars. (Use $\sqrt{3}$ = 1.73)



Sol. Let P and Q be two positions of the bird and the point of observation be A.

Given angles of elevation are

$$\angle PAB = 45°, \angle QAC = 30°$$

Given, height of tree PB = 80 m = QC

In right-angled ΔABP,

$$\tan 45° = \frac{BP}{AB}$$

$$\Rightarrow \quad 1 = \frac{80}{AB}$$

$$\Rightarrow \quad AB = 80 \text{ m}$$

In right-angled ΔACQ,

$$\tan 30° = \frac{CQ}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{AC}$$

$$\Rightarrow \quad AC = 80\sqrt{3} \text{ m}$$

$$\therefore \quad PQ = BC$$

$$= AC - AB = (80\sqrt{3} - 80) \text{ m}$$

$$= 80(\sqrt{3} - 1) \text{ m}$$

Therefore, bird flies $80(\sqrt{3}-1)$ m in 2 sec.

∴ Speed of the flying of the bird

$$= \frac{80(\sqrt{3}-1)}{2} = 40(\sqrt{3}-1) \text{ m/sec.}$$

$$\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$$

= 40 × 0.732 = 29.28 m/s.

Or

Sol. Let A, B be the cars approaching tower TW such that

$$\angle XTA = 30°, \angle XTB = 60°$$

$$\Rightarrow \quad \angle TAW = \angle XTA = 30°$$

$$\angle TBW = \angle XTB = 60°$$

Let $AB = x$, $BW = y$

In rt. $\angle d$ ΔTWB

$$\tan 60° = \frac{TW}{BW}$$

$$\Rightarrow \quad \sqrt{3} = \frac{75}{y}$$

$$\Rightarrow \quad y = \frac{75}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$y = 25\sqrt{3} \qquad \text{....}(i)$$

In rt. $\angle d$ ΔTWA

$$\tan 30° = \frac{TW}{AW}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{75}{x+y}$$

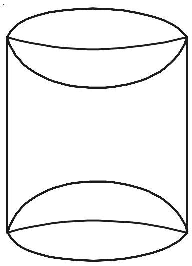
$$\Rightarrow \quad x + y = 75\sqrt{3} \qquad \ldots(ii)$$

$$(ii) - (i) \Rightarrow \; x + y - y = 75\sqrt{3} - 25\sqrt{3}$$

$$\Rightarrow \quad x = 50\sqrt{3} = 50 \times 1.73 = 86.5 \text{ m}$$

$\therefore$ Distance between cars is 86.5 m.

35. A wooden article was made by scooping out a hemisphere from each end of a salid cylinder, as shown in the figure. If the height of the cylinder is 5.8 cm and its bane is of radius 2.1 cm, find the total surface area of the article.



Or

There are two identical solid cubical boxes of side 7 cm. From the top face of the first cube, a hemisphere of diameter equal to the side of the cube is scooped out. This hemisphere is inverted and placed on the top of the second cube's surface to form a dome. Find

(*i*) the ratio of the total surface area of the two new solids formed

(*ii*)volume of each new solid formed.

Sol. Here, height of cylinder (h) = 5.8 cm

Radius of base of cylinder = Radius of hemisphere (r) = 2.1 cm

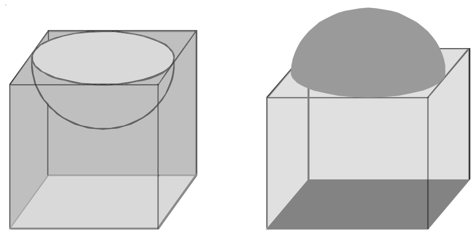
$\therefore$ Total surface area of wooden article = Curved surface area of cylinder + 2 (curved surface area of hemisphere)

$= 2\pi rh + 2\,(2\pi r^2) = 2\pi r\,(h + 2r)$

$$= 2 \times \frac{22}{7} \times 2.1 \times 10$$

$$= 132 \text{ cm}^2.$$

Or



First Solid Second Solid

(*i*) SA of first new solid (S_1) $= 6 \times a^2 + 2\pi r^2 - \pi r^2$

$= 6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2$

$= (294 + 77 - 38.5)$ cm^2

$= 332.5$ cm^2

SA of second new solid (S_2) $= 6 \times a^2 + 2\pi r^2 - \pi r^2$

$= 6 \times 7 \times 7 + 2\pi \times 3.5^2 - \pi \times 3.5^2$

$= 294 + 77 - 38.5 = 332.5$ cm^2

So, $S_1 : S_2 = 1 : 1$

(*ii*) Volume of first new solid (V_1) $= a^3 - \frac{2}{3}\pi r^3$

$$= 7 \times 7 \times 7 - \frac{2}{3}\pi \times 3.5^3$$

$$= 343 - \frac{539}{6} = \frac{1519}{6} \text{ cm}^3$$

Volume of second new solid (V_2)

$$= a^3 + \frac{2}{3}\pi r^3$$

$$= 7 \times 7 \times 7 + \frac{2}{3}\pi \times 3.5^3$$

$$= 343 + \frac{539}{6} = \frac{2597}{6} \text{ cm}^3.$$

SECTION—E

This section consists of 3 Case-Study Based Questions of 4 marks each.

36. It is common that governments revise travel fares from time to time based on various factors such as inflation (a general increase in prices and fall in the purchasing value of money) on different types of vehicles like auto, rickshaws, taxis, radio cab, etc. The auto charges in a city comprise a fixed charge (same for all the cities) together with the charge for the distance covered.

Name of the city	Distance travelled (km)	Amount paid (₹)

City A	10	75
	15	110
City B	8	91
	14	145

Situation 1 : In city A, for a journey of 10 km, the charge paid is (₹) 75 and for a journey of 15 km, the charge paid is (₹) 110.

Situation 2 : In a city B, for a journey of 8 km, the charge paid is (₹) 91 and for a journey of 14 km, the charge paid is (₹) 145.

Study the following situation and answer the following :

(*i*) Write the linear equation representing the situation for city A and B.

(*ii*) Find the running charges for both the cities and also find in which city auto fair is more.

(*iii*) Show the linear equation formed in Q.(*i*) for city B graphically.

Or

A person travelled 50 km, partially in city A and partially in city B. He travelled 30 km in city A and 20 km in city B. Find the amount paid by the person.

Sol. (*i*) For City A :

Let fixed charge is ₹ x and running charge is ₹ y/km.

$$x + 10y = 75 \quad ...(1)$$
$$x + 15y = 110 \quad ...(2)$$

For City B : Let running charge be ₹ p/km.

$$x + 8p = 91 \quad ...(3)$$
$$x + 14p = 145 \quad ...(4)$$

(*ii*) Solving equation for City A

(2) – (1), we have

$$5y = 35$$
$$y = 7$$

∴ Running charge is ₹ 7/km

for city B, solving both the equations

(4) – (3), we have

$$6p = 54$$
$$p = 9$$

∴ Running charge is city B is ₹ 9/km.

∴ Auto fair in city B is more than city A.

(*iii*) Let fixed charge be ₹ x and running charge is ₹ p/km.

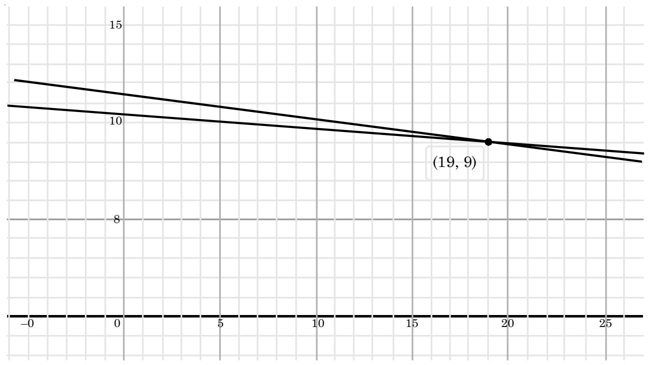
For city B, equations are

$$x + 8p = 91$$
$$x + 14p = 145$$

on solving, we get $p = 9$

$$x + 8 \times 9 = 91$$
$$x = 19$$

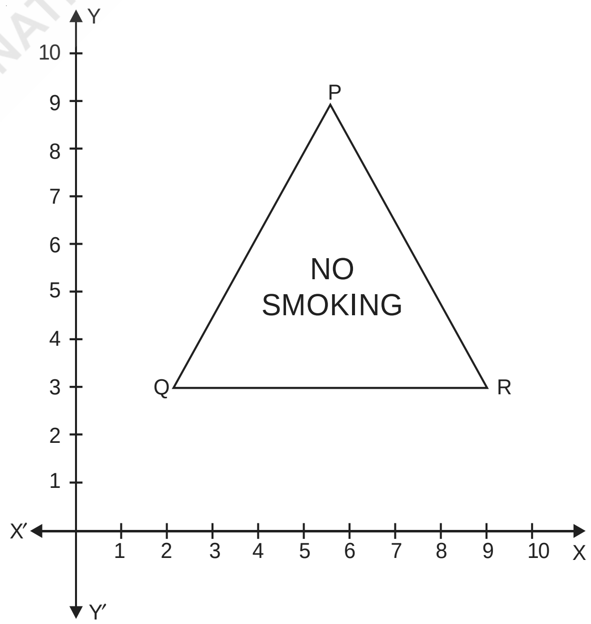
∴ fixed charge is ₹ 19.



Or

Total expenses $= (x + 30y) + (x + 20p)$
$= \{5 + 30(7)\} + \{19 + 20(9)\}$
$= 215 + 199 =$ ₹ 414

37. All of the persons know that smoking is injurious to health. So students of a school decided to make a campaign. To raise social awareness about hazards of smoking, school decided to start "NO SMOKING" campaign. Some students are asked to prepare campaign banners in the shape of traingles as shown in figure.



Answer the following questions :

(*i*) The co-ordinates of point P.

(*ii*) The co-ordinates of the mid point of Q and R.

(*iii*) The distance between P and R.

Or

The point on X-axis, which is equidistant from points Q and R.

Ans. (*i*) Co-ordinate of point P are (6,9)

(*ii*) Co-ordinate of the mid point of Q and

$$R = \left(\frac{2+9}{2}, \frac{3+3}{2}\right) = \left(\frac{11}{2}, 3\right)$$

(*iii*) $PR = \sqrt{(9-6)^2 + (3-9)^2} = \sqrt{(-3)^2 + (-6)^2}$

$= \sqrt{9+36} = \sqrt{45} = 3\sqrt{5}$ units

Or

Let A $(x, 0)$ be point on *x-axis* equidistant from Q and R.

$\therefore (AQ)^2 = (AR)^2$

$(x-2)^2 + (3-0)^2 = (x-9) + (3-0)^2$

$\Rightarrow x^2 - 4 - 4x + 9 = x^2 + 81 - 18x + 9$

$\Rightarrow -4x + 18x = 90 - 13 \Rightarrow 14x = 77 \Rightarrow x = \frac{77}{14} = \frac{11}{2}$

$\therefore$ co-ordinate of point A are $\left(\frac{11}{2}, 0\right)$

38. The following table shows the age distribution of case admitted during a day in a hospital.

Age (in years)	5–15	15–25	25–35	35–45	45–55	55–65
No. of cases	6	11	21	23	14	5

Answer the following questions :

(*i*) What is the upper limit of modal class ?

(*ii*) Find the mean of the given data.

(*iii*) What is the mode of the given data ?

OR

What is the median class ?

Ans. (*i*) Here modal class is 35-45

$\therefore$ upper limit of modal class is 45

(*ii*)

Age (in years)	No.of Cases (f)	*x*	*fx*	*c.f*
5-15	6	10	60	6
5-25	11	20	220	6+11=17
25-35	21	30	630	17+21=38
35-45	23	40	920	38+23=61
45-55	14	50	700	61+14=75
55-65	5	60	300	75+5=80
	$\Sigma f = 80$		$\Sigma fx = 2830$	

Mean $(\bar{x}) = \frac{\Sigma fx}{\Sigma f} = \frac{2830}{80} = 35.4$

(*iii*) Modal class is 35-45

$\therefore l = 35, f_1 = 23, fo = 21, f_2 = 14$ and R =10

Mode $= l + \left[\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right]$

$= 35 + \left[\frac{23-21}{2 \times 23 - 21 - 14}\right] \times 10 = 35 + \frac{20}{11} = 35 + 1.82 = 36$

Or

From the data table $\frac{N}{2} = \frac{80}{2} = 40$

$\therefore$ Medium Class is 35-45.

Holy Faith New Style Sample Paper–5

(Based on the Latest Design & Syllabus Issued by CBSE)

CLASS — 10th (CBSE)
MATHEMATICS (STANDARD)

Time Allowed : 3 Hours | **Maximum Marks : 80**

General Instructions : Same as in Holy Faith New Style Sample Paper—1.

SECTION—A

This section consists of 20 questions of 1 mark each.

1. The greatest number which divides 281 and 1249, leaving remainder 5 and 7 respectively, is :

(*a*) 23 (*b*) 276
(*c*) 138 (*d*) 69.

Ans. (*c*)

2. If α and β are the zeroes of a polynomial $f(x) = px^2 - 2x + 3p$ and $\alpha + \beta = \alpha\beta$, then p is :

(*a*) $-\frac{2}{3}$ (*b*) $\frac{2}{3}$
(*c*) $\frac{1}{3}$ (*d*) $-\frac{1}{3}$.

Ans. (*b*)

3. The sum and the product of zeroes of a quadratic polynomial are $-2\sqrt{3}$ and 3 respectively, there a quadratic polynomial is :

(*a*) $x^2 + 2\sqrt{3}x - 3$ (*b*) $(x - \sqrt{3})^2$
(*c*) $x^2 - 2\sqrt{3}x - 3$ (*d*) $x^2 + 2\sqrt{3}x + 3$

Ans. (*d*)

4. The system of equations $3x + y = 5$ and $6x + 2y = p$ are consistent, then value of p is :

(*a*) 5 (*b*) 8
(*c*) 2 (*d*) 10.

Ans. (*d*)

5. The roots of the quadratic equation $2x^2 - x - 6 = 0$ are :

(*a*) –2, 3/2 (*b*) 2, –3/2
(*c*) –2, –3/2 (*d*) 2, 3/2

Ans. (*b*)

6. The 21st term of the A.P., whose first two terms are –3 and 4 is :

(*a*) 143 (*b*) –143
(*c*) 17 (*d*) 137

Ans. (*d*)

7. The centre of a circle is at (2, 0). If one end of a diameter is at (6, 0), then the other end is at :

(*a*) (0, 0) (*b*) (4, 0)
(*c*) (– 2, 0) (*d*) (– 6, 0).

Ans. (*c*)

8. The ratio in which the line segment joining the points P(3, – 6) and Q(5, 3) divided by *x*-axis is :

(*a*) 1 : 3 (*b*) 2 : 1
(*c*) 3 : 2 (*d*) 1 : 4.

Ans. (*b*)

9. If $\tan\theta = \frac{2}{3}$, then the value of $\sec\theta$ is :

(*a*) $\frac{\sqrt{13}}{3}$ (*b*) $\frac{\sqrt{5}}{3}$
(*c*) $\frac{\sqrt{13}}{2}$ (*d*) $\frac{3}{\sqrt{13}}$.

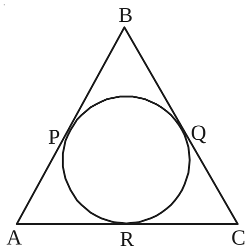
Ans. (*a*)

10. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then the value of $\sin^3\theta + \cos^3\theta$ is :

(*a*) 1 (*b*) $\frac{1}{2}$
(*c*) $\frac{\sqrt{2}}{2}$ (*d*) $\sqrt{2}$.

Ans. (*c*)

11. In the given figure, AB = BC = 10 cm. If AC = 7 cm, then the length of BP is :



(*a*) 3.5 cm (*b*) 7 cm
(*c*) 6.5 cm (*d*) 5 cm.

Ans. (*c*)

12. If the perimeter of a circle is equal to that of a square, then the ratio of their areas is :

(*a*) 22 : 7 (*b*) 14 : 11
(*c*) 7 : 22 (*d*) 11 : 14.

Ans. (*b*)

13. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. The ratio of their volumes is :

(*a*) 1 : 3 (*b*) 3 : 1
(*c*) 1 : 9 (*d*) 9 : 1

Ans. (*b*)

14. For the data 2, 9, $x + 6$, $2x + 3$, 5, 10, 5 ; if the mean is 7, then the value of x is :

(*a*) 9 (*b*) 6
(*c*) 5 (*d*) 3.

Ans. (*d*)

15. If the difference of mode and median of a data is 24, then the difference of median and mean is :

(*a*) 8 (*b*) 12
(*c*) 24 (*d*) 36

Ans. (*b*)

16. One ticket is drawn at random from a bag containing tickets numbered 1 to 40. The probability that the selected ticket has a number which is a multiple of 7 is :

(*a*) $\frac{1}{7}$ (*b*) $\frac{1}{8}$
(*c*) $\frac{1}{5}$ (*d*) $\frac{7}{40}$.

Ans. (*b*)

17. If an event that can not occurs then its probability is :

(*a*) 1 (*b*) $\frac{1}{2}$
(*c*) $\frac{3}{4}$ (*d*) 0.

Ans. (*d*)

18. Two dice are rolled. The probability of getting such numbers on two dice, whose product is perfect square is :

(*a*) $\frac{2}{9}$ (*b*) $\frac{1}{9}$
(*c*) $\frac{3}{8}$ (*d*) $\frac{1}{36}$.

Ans. (*a*)

Direction: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option.

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : If the second term of an A.P. is 13 and the fifth term is 25, then the 7th term is 33.

Reason (R) : If the common difference of an A.P. is 5, then $a_{18} - a_{13}$ is 25.

Sol. (*b*); In the given A.P., $a_2 = 13$ and $a_5 = 25$

$$a + d = 13 \quad ...(1)$$
$$a + 4d = 25 \quad ...(2)$$

Solving eqns. (1) and (2), we get $a = 9$ and $d = 4$

Thus, $a_n = a + (n - 1)d$

$a_7 = 9 + (7 - 1)4 = 33$

So, Assertion (A) is correct. In case of Reason (R),

In the given A.P., $d = 5$

So, $a_{18} - a_{13} = a + 17d - a - 12d$

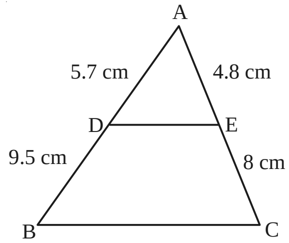
$= 5d = 25$

$\therefore$ Reason is correct.

Hence, both Assertion (A) and Reason (R) and correct but Reason (R) is not correct explanation of Assertion (A).

20. Assertion (A) : D and E are points on the sides AB and AC of ΔABC respectively, such that AD = 5.7 cm, DB = 9.5 cm, AE = 4.8 cm and EC = 8 cm, then DE $\nparallel$ BC.

Reason (R) : If a line divides any two sides of a triangle in same ratio, then it is parallel to the third side.



Sol.(*d*); $\frac{AD}{DB} = \frac{5.7}{9.5} = \frac{3}{5}$,

$\frac{AE}{EC} = \frac{4.8}{8} = \frac{3}{5}$

Since, $\frac{AD}{DB} = \frac{AE}{EC}$

$\therefore$ By converse of BPT, DE $\parallel$ BC.

So, Assertion (A) is false but, Reason (R) is true.

SECTION—B

This section consists of 5 questions of 2 marks each.

21. Show that the number 5 × 11 × 17 + 3 × 11 is a composite numbers.

Sol. We have :

$5 \times 11 \times 17 + 3 \times 11 = 11\,(85 + 3) = 11 \times 88$
$= 11 \times 11 \times 2 \times 2 \times 2$
$= 2 \times 2 \times 2 \times 11 \times 11$

So, it is a product of prime numbers.

Hence, it is a composite number.

22. If α and β are the zeroes of $4x^2 + 3x + 7$, then find the value of $\frac{1}{\alpha} + \frac{1}{\beta}$.

Or

If α and β are zeroes of a quadratic polynomial $x^2 - 25$, then form a quadratic polynomial whose zeroes are 1 + α and 1 + β.

Sol. Here, $p(x) = 4x^2 + 3x + 7$

$\therefore \quad \alpha + \beta = \frac{-b}{a} = \frac{-3}{4}$

and $\quad \alpha\beta = \frac{c}{a} = \frac{7}{4}$

Now $\quad \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{-3/4}{7/4} = -\frac{3}{7}$.

Or

Sol. Here $\quad p(x) = x^2 - 25$

$\therefore \quad x^2 - 25 = 0 \Rightarrow x^2 = 25 \Rightarrow x = \pm 5$

Let $\quad \alpha = 5$ and $\beta = -5$

$\therefore \quad 1 + \alpha = 1 + 5 = 6$ and $1 + \beta$
$= 1 + (-5) = 1 - 5 = -4$

So, 6 and – 4 are the zeroes of new quadratic polynomial

$\therefore$ New quadratic polynomial
$= x^2 - (6 - 4)\,x + (6 \times 4)$
$= x^2 - 2x - 24.$

23. Evaluate : $\frac{5}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cot^2 45° + 2 \sin^2 90°$.

Or

Prove that :

$$\sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}} = 2 \text{ cosec } A.$$

Sol. $\frac{5}{\cot^2 30°} + \frac{1}{\sin^2 60°} - \cot^2 45° + 2\sin^2 90°$

$$= \frac{5}{(\sqrt{3})^2} + \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} - (1)^2 + 2\,(1)^2$$

$$= \frac{5}{3} + \frac{1}{\left(\frac{3}{4}\right)} - 1 + 2 = \frac{5}{3} + \frac{4}{3} + 1$$

$$= \frac{5 + 4 + 3}{3} = \frac{12}{3} = 4$$

Or

Sol. $\text{LHS} = \sqrt{\frac{\sec A - 1}{\sec A + 1}} + \sqrt{\frac{\sec A + 1}{\sec A - 1}}$

$$= \frac{(\sqrt{\sec A - 1})^2 + (\sqrt{\sec A + 1})^2}{\sqrt{\sec^2 A - 1}}$$

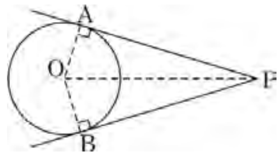
$$= \frac{(\sin A - 1) + (\sec A + 1)}{\sqrt{\tan^2 A}}$$

$$= \frac{2 \sec A}{\tan A} = \frac{\frac{2 \times 1}{\cos A}}{\frac{\sin A}{\cos A}} = 2 \text{ cosec } A = \text{RHS}$$

Hence proved

24. Prove that the lengths of tangents drawn from an external point to a circle are equal.

Sol.Given : PA and PB are tangents drawn to the circle with centre O from external point P.



To prove : PA = PB

Construction : Join OA, OB and OP.

Proof : $\angle OAP = \angle OBP = 90°$.

(Radius $\perp$ Tangent)

In ΔOAP and ΔOBP,

OA = OB (Radii of same circle)

OP = OP (Common side)

$\angle OAP = \angle OBP$ (Each 90°)

$\therefore$ $\Delta OAP \cong \Delta OBP$ (RHS congruency)

$\therefore$ PA = PB (*c.p.c.t.c.*)

Thus, lengths of tangents drawn from an external point are equal.

25. A die is thrown once. Find the probability of getting a number which (*i*) is a prime (*ii*) lies between 2 and 6.

Sol.(*i*) Here, total number of outcomes, n (S) = 6

Let E be the event of getting prime number, then E = {2, 3, 5}

Total number of favourable outcomes, n(E) = 3

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{6} = \frac{1}{2}$$

(*ii*) Let F be the event of getting a number between 2 and 6.

Then F = {3, 4, 5}

$\Rightarrow$ Total number of favourable outcomes, n(F) = 3

$$\therefore P(F) = \frac{n(F)}{n(S)} = \frac{3}{6} = \frac{1}{2}.$$

SECTION—C

This section consists of 6 questions of 3 marks each.

26. Prove that $\sqrt{5}$ is an irrational number.

Sol. Let, if possible $\sqrt{5}$ is not an irrational number. *i.e.*It is a rational number.

$\therefore \sqrt{5}$ can be expressed in $\frac{p}{q}$ form when p, q are coprime and $q \neq 0$.

$$\sqrt{5} = \frac{p}{q} \Rightarrow 5 = \frac{p^2}{q^2}$$

$\Rightarrow$ $p^2 = 5q^2$ ...(1)

$\Rightarrow$ 5 divides p^2

$\Rightarrow$ 5 divides p [$\because$ 5 is a prime number]

$\Rightarrow$ 5 is a factor of p ...(2)

So, let $p = 5\lambda$

Put $p = 5\lambda$ in (1)

$(5\lambda)^2 = 5q^2$

$\Rightarrow$ $25\lambda^2 = 5q^2$

$\Rightarrow$ $5\lambda^2 = q^2$

$\Rightarrow$ 5 divides q^2

$\Rightarrow$ 5 divides q

$\Rightarrow$ 5 is a factor of q ...(3)

From (2) and (3) $\Rightarrow$ 5 is a common factor of p and q which is a contradiction to the fact that p and q are coprime.

$\therefore$ Our supposition is wrong.

Hence, $\sqrt{5}$ is an irrational number.

27. A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.

Sol. Let the fraction be $\frac{x}{y}$

According to given conditions

$$\frac{x-1}{y} = \frac{1}{3} \text{ and } \frac{x}{y+8} = \frac{1}{4}$$

$\Rightarrow$ $3x - 3 = y$ and $4x = y + 8$

$\Rightarrow$ $3x - y - 3 = 0$ ...(1)

$4x - y - 8 = 0$ ...(2)

Subtracting (2) from (1)

$\Rightarrow$ $-x + 5 = 0 \Rightarrow x = 5$

Put $x = 5$ in (1), we get

$3(5) - y - 3 = 0$

$\Rightarrow$ $y = 15 - 3 \Rightarrow y = 12$

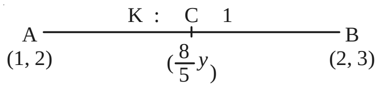
Required fraction is $\frac{5}{12}$

28. Find the ratio in which the point $\left(\frac{8}{5}, y\right)$ divides the line segment joining the points (1, 2) and (2, 3). Also, find the value of y.

Or

If the point P (x, y) is equidistant from the points A (a + b, b – a) and B (a – b, a + b). Prove that bx = ay.

Sol. (A) $\dfrac{2K+1}{K+1} = \dfrac{8}{5}$



$\Rightarrow \quad 10K + 5 = 8K + 8$

$\Rightarrow \quad 10K - 8K = 8 - 5$

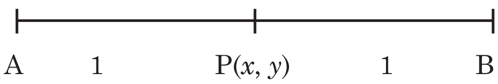
$\Rightarrow \quad 2K = 3 \Rightarrow K = \dfrac{3}{2}$

$\therefore$ Required ratio is 3 : 2

Also $\dfrac{9+4}{3+2} = y \Rightarrow y = \dfrac{13}{5}$.

Or

Sol. $(a + b, b - a)$ $(a - b, a + b)$



Using mid-point formula,

$$(x, y) = \left(\frac{a+b+a-b}{2}, \frac{b-a+a+b}{2}\right)$$

$\Rightarrow \quad x = \dfrac{a+b+a-b}{2} = a$ ....(*i*)

and $\quad y = \dfrac{b-a+a+b}{2} = b$ ....(*ii*)

(*i*) + (*ii*) gives, $\dfrac{x}{y} = \dfrac{a}{b}$

$\Rightarrow \quad bx = ay.$

29. Prove that :

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \csc\theta$$

Sol. $\text{LHS} = \dfrac{\tan\theta}{1-\cot\theta} + \dfrac{\cot\theta}{1-\tan\theta}$

$$= \frac{\tan\theta}{1-\dfrac{1}{\tan\theta}} + \frac{\dfrac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan\theta}{\left(\dfrac{\tan\theta - 1}{\tan\theta}\right)} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{-(1-\tan\theta)} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{1}{(1-\tan\theta)}\left[-\tan^2\theta + \frac{1}{\tan\theta}\right]$$

$$= \frac{1}{(1-\tan\theta)}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$$\frac{1}{(1-\tan\theta)} = \frac{(1-\tan\theta)(1+\tan^2\theta+\tan\theta)}{(\tan\theta)}$$

$$= \frac{(\sec^2\theta + \tan\theta)}{\tan\theta}$$

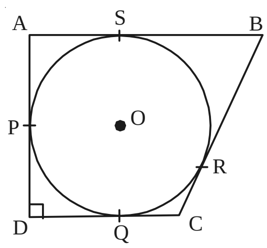
$$= \left(\frac{\sec^2\theta}{\tan\theta} + \frac{\tan\theta}{\tan\theta}\right)$$

$$= \frac{\sec\theta \times \cancel{\sec\theta}}{\sin\theta\,\cancel{\sec\theta}} + 1$$

$= \sec\theta + \csc\theta + 1$

= RHS. **(Hence proved)**

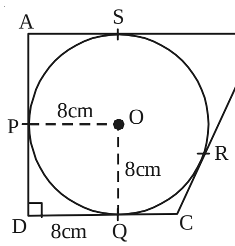
30. A circle with centre O and radius 8 cm is inscribed in a quadrilateral ABCD in which P, Q, R, S are the points of contact as shown. If AD is perpendicular to DC, BC = 30 cm and BS = 24 cm, then find the length DC.



Or

ABC is a right-angled triangle, right-angled at B and with BC = 6 cm and AB = 8 cm. A circle with centre O and radius *x* has been inscribed in ΔABC. Find the value of *x*.

Sol. We know that the tangent at any point of a circle is perpendicular to the radius through the point of contact.



$\therefore \Delta PDQ$ is a square

So $\quad OP = PD = DQ = OQ = 8$ cm

Also $\quad BS = BR = 24$ cm

[tangents drawn from an external point to a circle are equal]

$BC = BR + CR$

$\therefore \quad 30 = 24 + CR$

$\Rightarrow \quad CR = 30 - 24 = 6$ cm

Also $\quad CR = CQ = 6$ cm

[tangents drawn from an external point to a circle are equal]

Now $\quad DC = DQ + CQ$

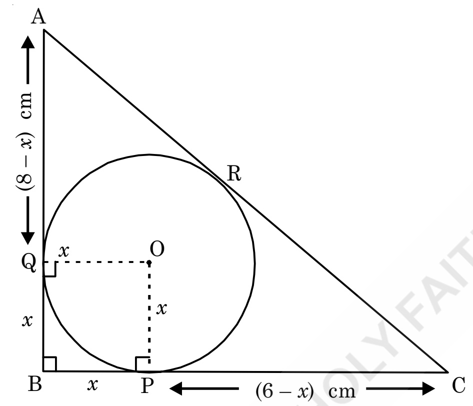
$= 8 + 6 = 14$ cm.

Or

Sol. We know that radius is perpendicular to tangent. $PO \perp BC$ and $OQ \perp AB$

$\therefore$ OPBQ is a rectangle.

But $OP = OQ = x$ (radii)



$\therefore$ OPBQ is a square.

Now, $\quad BP = x$

$\therefore \quad PC = (6 - x)$ cm

$\therefore \quad OP = QB = x$

$\therefore \quad AQ = AR = (8 - x)$ cm

and $\quad PC = CR = (6 - x)$ cm

Now, $\quad AC = \sqrt{AB^2 + BC^2}$

(By Pythagoras theorem)

$= \sqrt{8^2 + 6^2}$

$= \sqrt{64^2 - 36^2} = \sqrt{100} = 10$ cm

and $\quad AC = AR + CR$

$\Rightarrow \quad 10 = (8 - x) + (6 - x)$

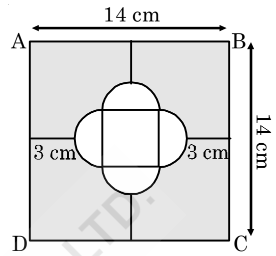
$\Rightarrow \quad 10 = 14 - 2x$

$\Rightarrow \quad 2x = 4$ cm

$\Rightarrow \quad x = 2$ cm

Thus, $\quad x = 2$ cm.

31. In figure, find the area of the shaded region, where ABCD is a square of side 14 cm in which four semicircles of same radii are drawn as shown. (Take π = 3.14)



Sol. Area of square $= 14 \times 14 = 196$ cm^2

Now let the radius of the circular quadrant $= r$

$\therefore r + r +$ side of white square $= 14 - 6 = 8$

$\Rightarrow \quad r + r + (2r) = 8$

$\Rightarrow \quad 4r = 8 \Rightarrow r = 2$

$\therefore$ Area of four semicircles $= 4\left(\frac{1}{2}\pi r^2\right) = 2\pi r^2$

$\Rightarrow \quad 2\pi\,(2)^2 = 8\pi$ cm^2

Side of white square $= 2r = 2 \times 2 = 4$ cm

$\therefore$Required area $= 196 - 8\pi - 16 = 180 - 8\pi$ cm^2

SECTION—D

This section consists of 4 questions of 5 marks each.

32. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with larger diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Sol. Let the smaller tap takes x hours to fill the tank.

$\therefore$The larger tap fills the tank in $(x - 2)$ hours.

Part of the tank filled by the smaller tap in 1 hour $= \frac{1}{x}$

Part of the tank filled by the larger tap in 1 hour $= \frac{1}{(x-2)}$

Time taken by both taps together to fill the tank

$= \frac{15}{8}$ hours

$\therefore$ Work done by both taps together in 1 hour $= \frac{8}{15}$

Therefore, $\frac{1}{x} + \frac{1}{x-2} = \frac{8}{15}$

$$\Rightarrow \frac{x-2+x}{x(x-2)} = \frac{8}{15} \Rightarrow \frac{2x-2}{x^2-2x} = \frac{8}{15}$$

$$\Rightarrow 30x - 30 = 8x^2 - 16x$$

$$\Rightarrow 8x^2 - 46x + 30 = 0$$

$$\Rightarrow 4x^2 - 23x + 15 = 0$$

$$\Rightarrow 4x^2 - 20x - 3x + 15 = 0$$

$$\Rightarrow 4x(x-5) - 3(x-5) = 0$$

$$\Rightarrow (4x-3)(x-5) = 0$$

$$\Rightarrow x = \frac{3}{4}, 5$$

($x = \frac{3}{4}$ is rejected as it gives $x - 2 = \frac{-5}{4}$ and time cannot be negative.)

$\therefore$ $x = 5$

Thus, the smaller and the larger taps can fill the tank separately in 5 hours and 3 hours respectively.

33. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find :

(*a*) The height of the basket from the ground.

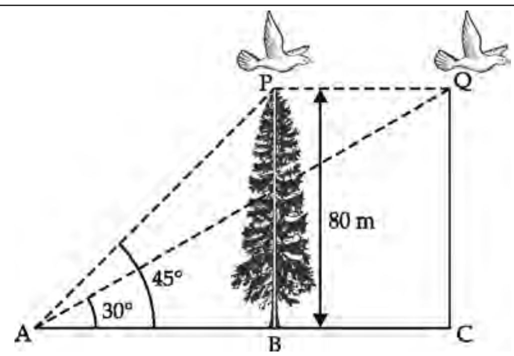
(*b*) The distance of the basket from the first observer's eye.

(*c*) The horizontal distance of the second observer from the basket.

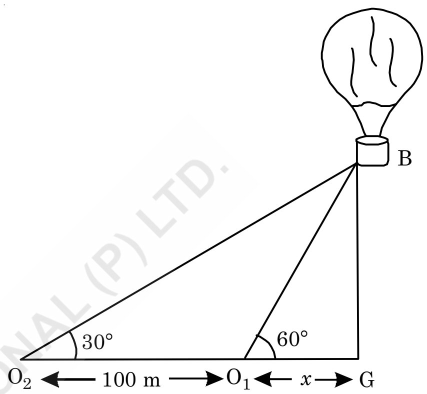
Or

A bird is sitting on the top of 80 m high tree. From a point on the ground, the angle of elevation of the bird is 45°. The bird flies away horizontally in such a way that it remained at a constant height from the ground. After 2 seconds, the angle of elevation of the bird from the same point is 30°. Find the speed of flying of the bird.

(Take $\sqrt{3}$ = 1.732)



Sol.



Let 'B' be basket of hot air balloon and O_1, O_2 be first and second observer respectively.

Angle of elevations are $\angle BO_1G = 60°$ and $\angle BO_2G = 30°$.

$$O_1O_2 = 100 \text{ m}$$

Let $O_1G = x$ m

(*a*) In rt. $\angle d$ ΔBGO_1,

$$\tan 60° = \frac{BG}{O_1G}$$

$$\Rightarrow \sqrt{3} = \frac{BG}{x} \Rightarrow BG = x\sqrt{3} \text{ m} \quad ....(i)$$

In rt. $\angle d$ ΔBGO_2,

$$\tan 30° = \frac{BG}{O_2G} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BG}{(x+100)}$$

$$\Rightarrow BG = \left(\frac{x+100}{\sqrt{3}}\right) \quad ...(ii)$$

Equating (*i*) and (*ii*),

$$x\sqrt{3} = \frac{x+100}{\sqrt{3}}$$

$\Rightarrow 3x = x + 100 \Rightarrow 2x = 100 \Rightarrow x = 50$ m.

$$O_1G = 50 \text{ m} \quad ...(iii)$$

Putting value of x in (*i*),

$$BG = 50\sqrt{3} \text{ m} \quad ...(iv)$$

So, height of basket from ground is $50\sqrt{3}$ m.

(*b*) Again in rt. $\angle d\ \Delta BGO_1$, $\sin 60° = \frac{BG}{O_1G}$

$$\Rightarrow \quad \frac{\sqrt{3}}{2} = \frac{50\sqrt{3}}{O_1B} \quad \text{(by using } (iv))$$

$$\Rightarrow \quad O_1B = 100 \text{ m.}$$

So, distance of first observer's eye from basket is 100 m.

(*c*) Horizontal distance of second observer from the basket = O_2G

$$= O_2O_1 + O_1G$$

$$= 100 + x = 100 + 50 \text{(by using } (iii))$$

$$= 150 \text{ m}$$

Or

Sol. Let P and Q be two positions of the bird and the point of observation be A.

Given angles of elevation are

$$\angle PAB = 45°, \angle QAC = 30°$$

Given, height of tree PB = 80 m = QC

In right-angled ΔABP,

$$\tan 45° = \frac{BP}{AB}$$

$$\Rightarrow \quad 1 = \frac{80}{AB}$$

$$\Rightarrow \quad AB = 80 \text{ m}$$

In right-angled ΔACQ,

$$\tan 30° = \frac{CQ}{AC}$$

$$\Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{80}{AC}$$

$$\Rightarrow \quad AC = 80\sqrt{3} \text{ m}$$

$$\therefore \quad PQ = BC$$

$$= AC - AB = (80\sqrt{3} - 80) \text{ m}$$

$$= 80(\sqrt{3} - 1) \text{ m}$$

Therefore, bird flies $80(\sqrt{3} - 1)$ m in 2 sec.

$\therefore$ Speed of the flying of the bird

$$= \frac{80(\sqrt{3} - 1)}{2} = 40(\sqrt{3} - 1) \text{ m/sec.}$$

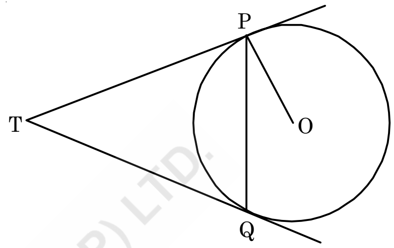
$$\left(\because \text{Speed} = \frac{\text{Distance}}{\text{Time}}\right)$$

$$= 40 \times 0.732 = 29.28 \text{ m/s.}$$

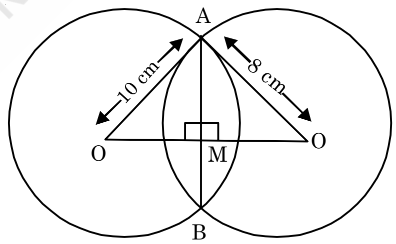
34. Two circles of radii 10 cm and 8 cm intersect and the length of the common chord is 12 cm. Find the distance between their centres. (Take $\sqrt{7}$ = 2.64)

Or

Two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that $\angle PTQ = 2\angle OPQ$.



Sol. Let the two circles having centre O and O′ and OA = 10 cm; O′A = 8 cm respectively.



Also, AB = 12 cm be the length of common chord

$$\therefore \quad AM = \frac{1}{2}AB$$

$$= \frac{1}{2}(12) = 6 \text{ cm}$$

In right-angled angled ΔOMA,

$$OA^2 = OM^2 + AM^2$$

$$\Rightarrow \quad (10)^2 = OM^2 + (6)^2$$

$$\Rightarrow \quad OM^2 = 100 - 36$$

$$\Rightarrow \quad OM^2 = 64 = (8)^2$$

$$\Rightarrow \quad OM = 8 \text{ cm}$$

Now, in right angled $\Delta O'MA$,

$$O'A^2 = O'M^2 + AM^2$$

$$\Rightarrow \quad (8)^2 = O'M^2 + (6)^2$$

$$\Rightarrow \quad O'M^2 = 64 - 36 = 28$$

$$\Rightarrow \quad O'M = \sqrt{28} = 5.29 \text{ cm}$$

$\therefore$ Required, distance between the centres

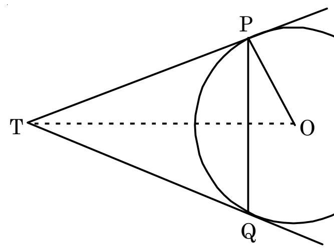
$= OO' = OM + MO'$

$= (8 + 5.29)$ cm

$= 13.29$ cm.

Or

Sol.



Let $\angle PTQ = \theta$

TPQ is an isosceles triangle.

$$\angle TPQ = \angle TQP$$

$$= \frac{1}{2}(180° - \theta)$$

$$= 90° - \frac{\theta}{2}$$

$$\angle OPT = 90°$$

$$\angle OPQ = \angle OPT - \angle TPQ$$

$$= 90° - \left(90° - \frac{\theta}{2}\right) = \frac{\theta}{2}$$

$$\angle OPQ = \frac{1}{2}\angle PTQ$$

$$2\angle OPQ = \angle PTQ$$

35. The following distribution gives the weights of 60 students of a class. Find the mean and mode weights of the students.

Weight (in kg)	No. of Students
40–44	4
44–48	6
48–52	10
52–56	14
56–60	10
60–64	8
64–68	6
68–72	**2**

Sol.

Weight (in kg)	No. of Students (f_i)	Mid-class (x_i)	$f_i x_i$
40–4	4	42	168
44–48	6	46	276
48–52	10	50	500
52–56	14	54	756
56–60	10	58	580
60–64	8	62	496
64–68	6	66	396
68–72	2	70	140
Total	$f_i = 60$		3312

$$\therefore \quad \text{Mean } (x) = \frac{\Sigma f_i x_i}{\Sigma f_i} = \frac{3312}{60} = 55.2$$

For mode :

$\therefore l = 52, f_1 = 14, f_0 = 10, f_2 = 10, h = 4$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_2 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 52 + \left(\frac{14 - 10}{2 \times 14 - 10 - 10}\right) \times 4$$

$$= 52 + \frac{4 \times 4}{28 - 20}$$

$$= 52 + \frac{16}{8} = 52 + 2 = 54.$$

SECTION—E

This section consists of 3 Case-Study Based Questions of 4 marks each.

36. Your elder brother wants to buy a car and plans to take a, loan from a bank for his car. He repays his total loan of ₹1,18,000 by paying every month starting with the first installment of ₹1000. He increases the installment by ₹100 every month.

Answer the following :

(*i*) The amount paid by him in 30th installment is :

(*ii*) The amount paid by him in 30 installments is :

(*iii*) What amount does he still have to pay after 30th installment ?

Or

If total installments are 40, then find the

amount paid in the last installment. Also, find the ratio of the 1st installment to the last installment.

Sol. (*i*) Various installments makes an A.P. 1000, 1100, 1200, 1300, ...

Here, $a = 1000, d = 100$

$\Rightarrow \quad a_{30} = a + 29d$

$= 1000 + 29\ (100) = ₹\ 3900$

(*ii*) Amount paid in 30 installments

$= S_{30} = \frac{30}{2}(2a + 29d)$

$= 15\ (2 \times 1000 + 29 \times 100) = 15 \times 4900 = ₹\ 73500$

(*iii*) Amount to be paid after 30th installment $= 118000 - S_{30} = 118000 - 73500 = ₹\ 44500$

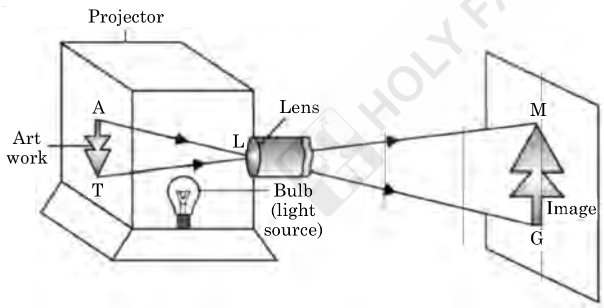
Or

Sol. Amount paid in last installment $= a_{40} = a + 39d$ $= 1000 + 39\ (100) = 4900$

Ratio of 1st installment to the last installment

$= \frac{1000}{4900} = \frac{10}{49} = 10 : 49$

37. An art projector is equipment used by artists to create exact copies of artwork, to enlarge artwork or to reduce artwork. A basic art projector used a light bulb and a lens within a box. The light rays from art work being copied are collected onto a lense at single point. The lens then projects the image of 'art work' onto the screen as shown.



Answer the following

(*i*) By which criteria ΔLAT ~ ΔLGM ?

(*ii*) If height of original artwork AT = 5 inches, AL = 15 inches and LG = 35 inches. Then find the height of the image.

(*iii*) If distance of screen from lense of projector is 20 inches, find the width of the 'art projector'.

Sol. (*i*) ∠ALT = ∠GLM [Vertically opposite ∠s]

∠ATL = ∠LMG [Alternate ∠*s*]

∠TAL = ∠MGL [Alternate ∠*s*]

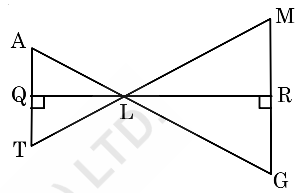
⇒ ΔALT ~ ΔGLM [By AAA similarity]

(*ii*) $\frac{AT}{GM} = \frac{AL}{GL}$ or $\frac{5}{GM} = \frac{15}{35}$

$\Rightarrow \quad GM = \frac{35}{3} = 11\frac{2}{3}$ inches.

(*iii*) Draw a line through L perpendicular to both AT as well as MG *i.e.* LQ ⊥ AT.

LR ⊥ MG.



In ΔAQL and ΔGRL,

$\Rightarrow \quad \frac{AL}{GL} = \frac{QL}{RL}$

(If Δs are similar, then ratio of their corresponding sides is same as ratio of their corresponding altitudes)

$\Rightarrow \quad \frac{15}{35} = \frac{QL}{20}$

$\Rightarrow \quad QL = \frac{15 \times 20}{35}$

$= \frac{60}{7} = 8\frac{4}{7}$ inches

38. In a wedding ceremony, cold-drink was served in cylinderical glasses and juice was served in conical glasses. The diameter and height of cylindrical glass are 7 cm and 6 cm respectively while diameter and height of conical glass are 8.4 cm and 12 cm respectively. On the basis of above information answer the following :

(*i*) Find the volume of cylinderical glass.

(*ii*) Find the volume of conical glass.

(*iii*) Find the surface area of cylinderical glass.

Or

Find the surface area of conical glass.

Sol. (*i*) Volume of cylinderical glass $= \pi r^2 h$

$= \frac{22}{7} \times 3.5 \times 3.5 \times 6 = 231\ cm^3$

(*ii*) Volume of conical glass $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \frac{22}{7} \times 4.2 \times 4.2 \times 12$

$= 221.76 \text{ cm}^3$

(*iii*) Surface area of cylinderical glass $= 2\pi rh + \pi r^2$

$= \pi r\,(2h + r)$

$= \frac{22}{7} \times 3.5\,(12 + 3.5)$

$= \frac{22}{7} \times 3.5 \times 15.5 = 170.5 \text{ cm}^2$

Or

Surface area of conical glass = πrl

$\therefore \quad l = \sqrt{h^2 + r^2} = \sqrt{(12)^2 + (4.2)^2}$

$= \sqrt{144 + 17.64} = \sqrt{161.64}$

$= 12.71 \text{ cm}$

$\therefore \quad S = \frac{22}{7} \times 4.2 \times 12.71 = 167.77 \text{ cm}^2.$

Holy Faith New Style Sample Paper–6

(Based on the Latest Design & Syllabus Issued by CBSE)

CLASS — 10th (CBSE)
MATHEMATICS (STANDARD)

Time Allowed : 3 Hours **Maximum Marks : 80**

General Instructions : Same as in Holy Faith New Style Sample Paper—1.

SECTION—A

This section consists of 20 questions of 1 mark each.

1. Three alarm clocks ring their alarms at regular intervals of 20 min, 25 min and 30 min respectively. If they first beep together at 12 noon, at what time will they beep again for the first time ?

(*a*) 4:00 pm (*b*) 4:30 pm
(*c*) 5:00 pm (*d*) 5:30 pm.

Ans. (*c*)

2. The number of polynomial having zeroes – 3 and 5 is :

(*a*) only one (*b*) infinite
(*c*) exactly two (*d*) atmost two.

Ans. (*b*)

3. If a pair of linear equations inconsistent then the lines will be :

(*a*) parallel
(*b*) always coincident
(*c*) intersecting or coincident
(*d*) always intersecting.

Ans. (*c*)

4. If the pair of equations $2x + 3y = 7$ and $kx + \frac{9}{2}y = 12$ have no solution, then the value of k is :

(*a*) $\frac{2}{3}$ (*b*) $\frac{3}{2}$
(*c*) 3 (*d*) – 3.

Ans. (*c*)

5. The value of *k* for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other is :

(*a*) – 3 (*b*) $\frac{1}{3}$
(*c*) 3 (*d*) 9.

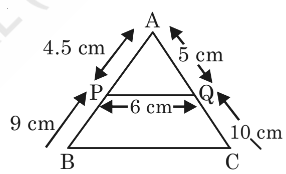
Ans. (*c*)

6. The number of terms in the A.P. 3, 6, 9, 12, ... , 111 is :

(*a*) 36 (*b*) 40
(*c*) 37 (*d*) 30.

Ans. (*c*)

7. In fig., P and Q are points on the sides AB and AC respectively of ΔABC such that AP = 4.5 cm, PB = 9 cm, AQ = 5 cm, QC = 10 cm. If PQ = 6 cm, then BC is equal to :



(*a*) 12 cm (*b*) 18 cm
(*c*) 24 cm (*d*) 30 cm.

Ans. (*b*)

8. If the distance between the points (3, – 5) and (x, – 5) is 15 units, then the values of x are :

(*a*) 12, – 18 (*b*) – 12, 18
(*c*) 18, 5 (*d*) – 9, – 12.

Ans. (*b*)

9. The point P which divides the line segment joining the points A(2, –5) and B(5, 2) in the ratio 2 : 3 lies in the quadrant :

(*a*) I (*b*) III
(*c*) IV (*d*) None of these.

Ans. (*c*)

10. If $0° \leq A, B \leq 45°$ such that sin (A + B) = 1 and cos (A – B) = $\frac{\sqrt{3}}{2}$, the values of A and B respectively are :

(*a*) 60°, 30° (*b*) 30°, 60°
(*c*) 30°, 45° (*d*) 45°, 60°.

Ans. (*a*)

11. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$, then the angle of elevation of the sun is :

(*a*) 30° (*b*) 60°
(*c*) 15° (*d*) 75°.

Ans. (*a*)

12. The diameter of a car wheel is 42 cm. The number of complete revolutions it will make in moving 1.32 km is :

(*a*) 10^4 (*b*) 10^5
(*c*) 10^{-8} (*d*) 10^3.

Ans. (*b*)

13. A solid sphere is cut into two hemisphere. The ratio of the surface areas of sphere to that of two hemispheres taken together is :

(*a*) 1 : 1 (*b*) 1 : 4
(*c*) 2 : 3 (*d*) 3 : 2.

Ans. (*c*)

14. For the following distribution :

C.I.	0-5	5-10	10-15	15-20	20-25
Frequency	10	15	12	20	9

the sum of the lower limits of the median and modal class is :

(*a*) 15 (*b*) 25
(*c*) 30 (*d*) 35

Ans. (*b*)

15. If every term of the statistical data consisting of n terms of decreased by 2, then the mean of the data :

(*a*) decreases by 2 (*b*) remains unchanged
(*c*) decreases by 2*n* (*d*) decreases by 1.

Ans. (*a*)

16. If the probability of a player winning a game is 0.79, then the probability of his losing the game is :

(*a*) 1.79 (*b*) 0.31
(*c*) 0.21% (*d*) 0.21.

Ans. (*d*)

17. From the data 1, 4, 7, 9, 16, 21, 25, if all the even numbers are removed, then the probability of getting at random a prime number from the remaining is :

(*a*) $\frac{2}{5}$ (*b*) $\frac{1}{5}$

(*c*) $\frac{1}{7}$ (*d*) $\frac{2}{7}$.

Ans. (*b*)

18. A card is drawn from a well-shuffed deck of 52 playing cards. The probability that the card will not be an ace is :

(*a*) $\frac{1}{13}$ (*b*) $\frac{1}{4}$

(*c*) $\frac{12}{13}$ (*d*) $\frac{3}{4}$.

Ans. (*c*)

Direction: In the question number 19 and 20, a statement of **Assertion (A)** is followed by a statement of **Reason (R)**.

Choose the correct option.

(*a*) Both assertion (A) and reason (R) are true and reason (R) is the correct explanation of assertion (A).

(*b*) Both assertion (A) and reason (R) are true and reason (R) is not the correct explanation of assertion (A).

(*c*) Assertion (A) is true but reason (R) is false.

(*d*) Assertion (A) is false but reason (R) is true.

19. Assertion (A) : The system $kx - y = 2$ and $6x - 2y = 3$ has a unique solutions only when $k = 3$.

Reason (R) : $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ has unique solution if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$.

Sol. (*d*);
$$kx - y = 2 \quad ...(1)$$
$$6x - 2y = 3 \quad ...(2)$$

For unique solution,
$$\frac{k}{6} \neq \frac{1}{2}$$
$$\Rightarrow \quad k \neq 3$$

∴ Assertion is false but Reason is true.

20. Assertion (A) : The length of the tangent drawn from a point 8 cm away from the centre of the circle of radius 6 cm is $2\sqrt{7}$ cm

Reason (R) : If the angle between two radii of a circle is 130°, then the angle between the tangents is 50°.

Sol. (*b*); In case of Assertion (A) :

$$\text{Length of the tangent} = \sqrt{d^2 - r^2} = \sqrt{(8)^2 - (6)^2}$$
$$= \sqrt{64 - 36} = \sqrt{28}$$
$$= 2\sqrt{7}$$

∴ Assertion is correct.

In case of Reason (R) : Since, sum of the angles between radii and angle between the tangents is 180°.

Angle between the tangents = 180° – 130° = 50°

∴ Reason (R) is correct.

Hence both Assertion (A) and Reason (R) are correct but reason is not the correct explanation for Assertion (A).

SECTION—B

This section consists of 5 questions of 2 marks each.

21. If the HCF of 65 and 117 is expressible in the form 65m – 117, then find value of m.

OR

Two numbers are in the ratio 2 : 3 and their LCM is 180. What is the HCF of these numbers ?

Sol.
$$65 = 5 \times 13$$
$$117 = 3^2 \times 13$$
$$\text{HCF} = 13$$
$$\therefore \quad 65 \times m - 117 = 13$$
$$\Rightarrow \quad 65m = 130$$
$$\Rightarrow \quad m = 130 \div 65$$
$$\Rightarrow \quad m = 2$$

Or

Let the numbers be $2x$ and $3x$.
$$\therefore \quad \text{LCM} = 2 \times 3 \times x = 6x$$
$$\Rightarrow \quad 6x = 180$$
$$\Rightarrow \quad x = 30$$

Thus, numbers are 2 × 30, 3 × 30 *i.e.* 60, 90

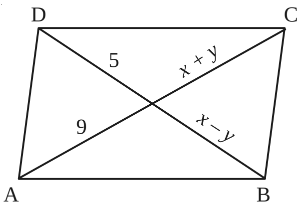
HCF (a, b) × LCM $(a, b) = a \times b$
$$\Rightarrow \quad \text{HCF} \times 180 = 60 \times 90$$
$$\text{HCF} = \frac{60 \times 90}{180} = 30$$

22. For what value of p will the following pair of linear equations have infinitely many solutions.

$$(p-3)\,x + 3y = p$$
$$px + py = 12.$$

OR

In the given figure ABCD is a parallelogram. Find the values of x and y.



Sol. For infinitely many solutions,
$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{p-3}{p} = \frac{3}{p} = \frac{p}{12}$$

First two ratios ⇒ $(p - 3)\, p = 3p$
$$\Rightarrow \quad p^2 - 6p = 0$$
$$\Rightarrow \quad p\,(p-6) = 0$$
$$\Rightarrow \quad p = 0, 6 \quad \text{...(1)}$$

Last two ratios
$$\Rightarrow \quad p^2 = 36$$
$$\Rightarrow \quad p = \pm 6 \quad \text{...(2)}$$

(1) and(2) agree on $p = 6$
$$\therefore \quad p = 6.$$

Or

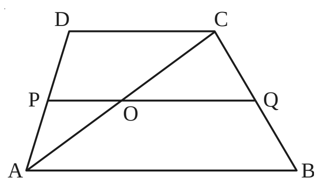
Sol. We know that diagonals of a parallelogram bisect each other.
$$\therefore \quad x + y = 9 \quad \text{....}(i)$$
$$\text{and} \quad x - y = 5 \quad \text{....}(ii)$$

Adding equations (*i*) and (*ii*), we get :
$$x + y + x - y = 9 + 5 \Rightarrow 2x = 14 \Rightarrow x = 7$$

Putting value of x in equation (*i*), we get
$$7 + y = 9 \Rightarrow y = 9 - 7 \Rightarrow y = 2$$

Hence, $x = 7$ and $y = 2$.

23. ABCD is a trapezium in which AB | | DC. P and Q are points on sides AD and BC respectively such that PQ | | AB. If PD = 18 cm, EQ = 35 cm and QC = 15 cm. Find the value of AD.



Sol. In ΔABC, OQ | | AB
$$\therefore \quad \frac{\text{AO}}{\text{OC}} = \frac{\text{BQ}}{\text{QC}} \quad \text{....}(i)$$

In ΔACD, OP | | DC
$$\therefore \quad \frac{\text{AO}}{\text{OC}} = \frac{\text{AP}}{\text{PD}} \quad \text{....}(ii)$$

From equations (*i*) and (*ii*), we get :
$$\frac{\text{AP}}{\text{PD}} = \frac{\text{BQ}}{\text{QC}}$$
$$\therefore \quad \frac{\text{AP}}{18} = \frac{35}{15}$$
$$\Rightarrow \quad \text{AP} = \frac{18 \times 35}{15} = 42 \text{ cm}$$
$$\therefore \quad \text{AD} = \text{AP} + \text{PD}$$
$$= 42 + 18 = 60 \text{ cm.}$$

24. 11. If $7\sin^2\theta + 3\cos^2\theta = 4$, then prove that $\sec\theta + \text{cosec}\,\theta = 2 + \frac{2}{\sqrt{3}}$.

Sol. Given : $7\sin^2\theta + 3\cos^2\theta = 4$

Dividing both sides by $\cos^2\theta$, we get

$\Rightarrow \quad 7\tan^2\theta + 3 = 4\sec^2\theta$

$\Rightarrow \quad 7(\sec^2\theta - 1) + 3 = 4\sec^2\theta$

$\Rightarrow 7\sec^2\theta - 4\sec^2\theta = 4$

$\Rightarrow \quad 3\sec^2\theta = 4$

$\Rightarrow \quad \sec\theta = \sqrt{\frac{4}{3}} = \frac{2}{\sqrt{3}} \qquad ...(i)$

$\Rightarrow \quad \cos\theta = \frac{\sqrt{3}}{2}$

Now, $\quad \sin\theta = \sqrt{1-\cos^2\theta}$

$\Rightarrow \quad \sin\theta = \sqrt{1-\frac{3}{4}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

and $\quad \text{cosec}\,\theta = \frac{1}{\sin\theta} = 2 \qquad ...(ii)$

(i) and $(ii) \Rightarrow \sec\theta + \text{cosec}\,\theta = 2 + \frac{2}{\sqrt{3}}$ **Q.E.D.**

25. Ages of the pumps manufactured by a company are given in the following distribution table. Calculate the modal age of pumps.

Age (in years)	No. of pumps
0 – 1	12
1 – 2	18
2 – 3	45
3 – 4	23
4 – 5	12
5 – 6	08
6 – 7	04

Sol. $\therefore$ Maximum frequency 45 corresponding to C.I.2–3.

$\therefore$ Modal class = 2 – 3.

Here, $l = 2, f_1 = 45, f_0 = 18, f_2 = 23, h = 1$

$$\therefore \quad \text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)\times h$$

$$= 2 + \left(\frac{45-18}{2\times 45 - 18 - 23}\right)\times 1$$

$$= 2 + \left(\frac{27}{90-41}\right)$$

$$= 2 + \frac{27}{49}$$

$$= 2 + 0.55 = 2.55.$$

SECTION—C

This section consists of 6 questions of 3 marks each.

26. Given that $\sqrt{3}$ is irrational, prove that $5+2\sqrt{3}$ is irrational.

Given that $\sqrt{3}$ is irrational, prove that $5+2\sqrt{3}$ is irrational.

Sol. Let us assume $5+2\sqrt{3}$ is rational, then it must be in the form of $\frac{p}{q}$ where p and q are co-prime integers and $q \neq 0$

i.e. $\quad 5+2\sqrt{3} = \frac{p}{q}$

So, $\quad \sqrt{3} = \frac{p-5q}{2q} \qquad ....(i)$

Since p, q, 5 and 2 are integers and $q \neq 0$. RHS of equation (i) is rational. But LHS of (i) is $\sqrt{3}$ which is irrational. This is not possible.

This contradiction has arisen due to our wrong assumption that $5+2\sqrt{3}$ is rational. So $5+2\sqrt{3}$ is irrational.

27. Three consecutive natural numbers are such that the square of the middle number exceeds the difference of the square of other two by 60. Find the numbers.

OR

If $ad \neq bc$, then prove that the equation $(a^2 + b^2)x^2 + 2(ac + bd)x + (c^2 + d^2) = 0$ has no real roots.

Sol. Let three consecutive natural numbers be $x, x+1, x+2$, then

$(x+1)^2 = 60 + [(x+2)^2 - x^2]$

$\Rightarrow \quad x^2 + 1 + 2x = 60 + [x^2 + 4 + 4x - x^2]$

$\Rightarrow \quad x^2 + 1 + 2x = 64 + 4x$

$\Rightarrow \quad x^2 - 2x - 63 = 0$

$\Rightarrow \quad x^2 - 9x + 7x - 63 = 0$

$\Rightarrow \quad x(x-9) + 7(x-9) = 0$

$\Rightarrow \quad (x-9)(x+7) = 0 \Rightarrow x = 9, -7$

But x is a natural number.

$\therefore \quad x = 9$

Hence, the required numbers are 9, 10, 11.

Or

Sol. We know that, $\quad D = B^2 - 4AC$

Here, $\quad A = (a^2 + b^2)$,

$B = 2(ac + bd), C = (c^2 + d^2)$

Thus, $D = B^2 - 4AC$

$= [2(ac + bd)]^2 - 4(a^2 + b^2)(c^2 + d^2)$

$= 4[a^2c^2 + b^2d^2 + 2abcd] - 4[a^2c^2 + a^2d^2 + b^2c^2 + b^2d^2]$

$= -4[a^2d^2 - b^2c^2 - 2abcd]$

$= -4[ad - bc]^2$

Since, $ad \neq bc$ (Given)

$\therefore \quad (ad - bc)^2 > 0$

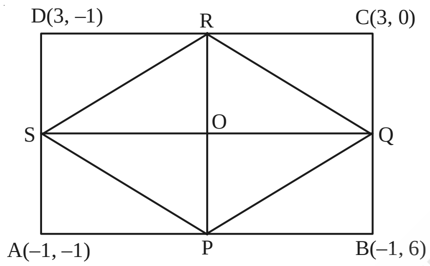
$\Rightarrow \quad -4(ad - bc)^2 < 0$

$\therefore \quad < 0$

Hence, given equation has no real roots.

28. ABCD is a rectangle formed by the points A (– 1, – 1), B (– 1, 6), C (3, 6) and D (3, – 1). P, Q, R and S are mid points of sides AB, BC, CD and DA respectively. Show that the diagonals of the quadrilateral PQRS bisect each other.

Sol. (B) Since P, Q, R and S are mid-points of sides AB, BC, CD and DA respectively.



$\therefore$ Co-ordinates of point P

$$= \left(\frac{-1-1}{2}, \frac{-1+6}{2}\right) = \left(-1, \frac{5}{2}\right)$$

Co-ordinates of point

$$Q = \left(\frac{-1+3}{2}, \frac{6+6}{2}\right) = (1, 6)$$

Co-ordinates of point

$$R = \left(\frac{3+3}{2}, \frac{6-1}{2}\right) = \left(3, \frac{5}{2}\right)$$

Co-ordinates of point

$$S = \left(\frac{3-1}{2}, \frac{-1-1}{2}\right) = (1, -1)$$

Now $$PQ = \sqrt{(-1-1)^2 + \left(\frac{5}{2} - 6\right)^2}$$

$$= \sqrt{4 + \frac{49}{4}} = \frac{\sqrt{65}}{2}$$

$$QR = \sqrt{(1-3)^2 + \left(6 - \frac{5}{2}\right)^2} = \sqrt{4 + \frac{49}{4}} = \frac{\sqrt{65}}{2}$$

$$RS = \sqrt{(3-1)^2 + \left(\frac{5}{2} + 1\right)^2} = \sqrt{4 + \frac{49}{4}} = \frac{\sqrt{65}}{2}$$

$$SP = \sqrt{(1+1)^2 + \left(-1 - \frac{5}{2}\right)^2} = \sqrt{4 + \frac{49}{4}} = \frac{\sqrt{65}}{2}$$

$$PR = \sqrt{(-1-3)^2 + \left(\frac{5}{2} - \frac{5}{2}\right)^2} = \sqrt{(4)^2} = 4$$

$$QS = \sqrt{(1-1)^2 + (6+1)^2} = \sqrt{(7)^2} = 7$$

$\therefore$ PQ = QR = RS = SP but PR $\neq$ QS

Which shows that PQRS is a rhombus and we know that diagonals of a rhombus bisect each other.

29. Prove that :

$$\frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta} = 1 + \sec\theta \operatorname{cosec}\theta$$

Or

Prove that : $$\cot\theta - \tan\theta = \frac{2\cos^2\theta - 1}{\sin\theta\cos\theta}.$$

Sol. $$\text{LHS} = \frac{\tan\theta}{1-\cot\theta} + \frac{\cot\theta}{1-\tan\theta}$$

$$= \frac{\tan\theta}{1 - \frac{1}{\tan\theta}} + \frac{\frac{1}{\tan\theta}}{1-\tan\theta}$$

$$= \frac{\tan\theta}{\left(\frac{\tan\theta - 1}{\tan\theta}\right)} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{\tan^2\theta}{-(1-\tan\theta)} + \frac{1}{\tan\theta(1-\tan\theta)}$$

$$= \frac{1}{(1-\tan\theta)}\left[-\tan^2\theta + \frac{1}{\tan\theta}\right]$$

$$= \frac{1}{(1-\tan\theta)}\left[\frac{1-\tan^3\theta}{\tan\theta}\right]$$

$[\because a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$

$$\frac{1}{(1-\tan\theta)} = \frac{(1-\tan\theta)(1+\tan^2\theta+\tan\theta)}{(\tan\theta)}$$

$$= \frac{(\sec^2\theta+\tan\theta)}{\tan\theta}$$

$$= \left(\frac{\sec^2\theta}{\tan\theta}+\frac{\tan\theta}{\tan\theta}\right)$$

$$= \frac{\sec\theta\times\cancel{\sec\theta}}{\sin\theta\,\cancel{\sec\theta}}+1$$

$= \sec\theta + \operatorname{cosec}\theta + 1$

= RHS. **(Hence proved)**

Or

Sol. L.H.S.= $\cot\theta - \tan\theta$

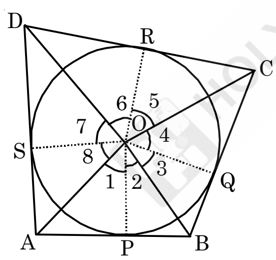
$$= \frac{\cos\theta}{\sin\theta} - \frac{\sin\theta}{\cos\theta} = \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta}$$

$$= \frac{\cos^2\theta-(1-\cos^2\theta)}{\sin\theta\cos\theta} = \frac{2\cos^2\theta-1}{\sin\theta\cos\theta}$$

= R.H.S. **Q.E.D.**

30. Prove that opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

Sol.Given : A circle with centre O touches the sides AB, BC, CD and DA of a quadrilateral ABCD at points P, Q, R and S respectively.



To Prove : $\angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

Construction : Join OP, OQ, OR and OS.

Proof. Since the two tangents drawn from an external point to a circle subtend equal angles at the centre.

$\therefore \angle 2 = \angle 3, \angle 4 = \angle 5, \angle 6 = \angle 7, \angle 8 = \angle 1$ ...(1)

Now, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 + \angle 6 + \angle 7 + \angle 8 = 360°$

($\because$ Sum of all the angles subtended at a point is 360°.)

$\Rightarrow 2(\angle 1 + \angle 2 + \angle 5 + \angle 6) = 360°$

and $2(\angle 3 + \angle 4 + \angle 7 + \angle 8) = 360°$

$\Rightarrow (\angle 1 + \angle 2) + (\angle 5 + \angle 6) = 180°$

and $(\angle 3 + \angle 4) + (\angle 7 + \angle 8) = 180°$

($\because \angle 1 + \angle 2) = \angle AOB, (\angle 5 + \angle 6) = \angle COD$, $\angle 7 + \angle 8 = \angle AOD$ and $\angle 3 + \angle 4 = \angle BOC$)

$\Rightarrow \angle AOB + \angle COD = 180°$ and $\angle AOD + \angle BOC = 180°$

Thus, opposite sides of a quadrilateral circumscribing a circle subtend supplementary angles at the centre of the circle.

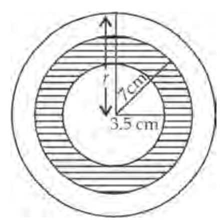
31. Find the area enclosed between two concentric circles of radii 3.5 cm, 7 cm. A third concentric circle is drawn outside the 7 cm circle so that the area enclosed between it and the 7 cm circle is same as that between two inner circles. Find the radius of the third circle.

Sol. Area between first two circles

$= \pi \times 7^2 - \pi \times 3.5^2 = 49\pi - 12.25\pi$ ... (1)

Area between next two circles

$= \pi r^2 - \pi \times 7^2 = \pi r^2 - 49\pi$ ...(2)



According to question, (1) and (2) are equal.

$49\pi - 12.25\pi = \pi r^2 - 49\pi$

$\Rightarrow \pi r^2 = 49\pi + 49\pi - 12.25\pi$

$\Rightarrow r^2 = 98 - 12.25 = 85.75$

$\Rightarrow r^2 = \frac{8575}{100} = \frac{343}{4}$

$\Rightarrow r = \frac{\sqrt{343}}{2}$ cm

From (1) $\Rightarrow$ Area between first two circles

$= 49\pi - 12.25\pi$

$= \frac{22}{7}(49 - 12.25)$

$= \frac{22}{7} \times 36.75$

$= 22 \times 5.25 = 115.5$ cm^2.

SECTION—D

This section consists of 4 questions of 5 marks each.

32. If the sum of the first p terms of an A.P. is q and the sum of the first q terms is p; then show that sum of $(p + q)$ terms is $\{-(p + q)\}$.

Or

The sum of four consecutive numbers in A.P. is 32 and the ratio of the product of the first and the last terms to the product of two middle terms is 7 : 15. Find the numbers.

Sol. Let a, d be first term and common difference of given A.P., respectively.

Given that

$$S_p = q$$

$$\Rightarrow \frac{p}{2}[2a + (p-1)d] = q$$

$$\Rightarrow 2ap + (p^2 - p)d = 2q \quad ...(1)$$

$$S_q = p$$

$$\Rightarrow \frac{q}{2}[2a + (q-1)d] = p$$

$$\Rightarrow 2aq + (q^2 - q)d = 2p \quad ...(2)$$

Subtracting equation (2) from equation (1)

$\Rightarrow 2a(p-q) + (p^2 - p - q^2 + q)\,d = 2(q-p)$

$\Rightarrow 2a(p-q) + [(p^2 - q^2) - (p-q)]\,d = 2(q-p)$

$\Rightarrow 2a(p-q) + [(p-q)(p+q) - (p-q)]\,d = 2(q-p)$

$\Rightarrow (p-q)\,[2a + (p+q-1)\,d\,] = 2(q-p)$

$\Rightarrow 2a + (p+q-1)d = -2 \quad ...(3)$

$$\therefore S_{p-q} = \frac{p+q}{2}[2a + (p+q-1)d] = \frac{p+q}{2}(-2)$$

[Using eqn. (3)]

$= -(p+q)$ **Q.E.D.**

Or

Sol. Let four consecutive number in A.P. be $a - 3d$, $a - d$, $a + d$, $a + 3d$.

Sum $= a - 3d + a - d + a + d + a + 3d = 32$

$\Rightarrow 4a = 32 \Rightarrow a = 8$

$\therefore$ Numbers are $8 - 3d$, $8 - d$, $8 + d$, $8 + 3d$

Given : $$\frac{(8-3d)(8+3d)}{(8-d)(8+d)} = \frac{7}{15}$$

$$\Rightarrow \frac{64 - 9d^2}{64 - d^2} = \frac{7}{15}$$

$$\Rightarrow 960 - 135d^2 = 448 - 7d^2$$

$$\Rightarrow 512 = 128d^2$$

$$\Rightarrow d^2 = \frac{512}{128} = 4$$

$$d = \pm 2$$

When $d = 2$; numbers are $8 - 3\,(2)$, $8 - 2$, $8 + 2$, $8 + 3\,(2)$

or 2, 6, 10, 14

When $d = -2$, numbers are 14, 10, 6, 2.

Thus, the required numbers are 2, 6, 10, 14.

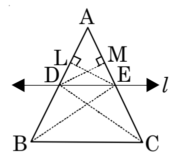
33. Prove that if a line is drawn parallel to one side of a triangle, the other two sides are divided in the same ratio.

Using the above result, show that diagonals of a trapezium cut each other in the same ratio.

Sol. First Part :

Given : ΔABC and line 'l' parallel to BC intersect AB at D and AC at E as shown below :

To Prove : $$\frac{AD}{DB} = \frac{AE}{EC}$$



Construction : Join BE and CD and draw EL $\perp$ AB and DM $\perp$ AC.

Proof : Area of ΔADE $= \frac{1}{2}(AD \times EL)$

Area of ΔBDE $= \frac{1}{2}(BD \times EL)$

$$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta BDE} = \frac{\frac{1}{2}(AD \times EL)}{\frac{1}{2}(BD \times EL)} = \frac{AD}{DB} \quad ...(1)$$

Similarly,

$$\frac{\text{Area of } (\Delta ADE)}{\text{Area of } (\Delta CDE)} = \frac{\frac{1}{2} AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \quad ...(2)$$

But, Area (ΔBDE) = Area (ΔCDE) ...(3)

(The two areas are equal because the two triangles are on the same base DE and between the same parallel lines DE and BC.)

Using (3) in (2), we get

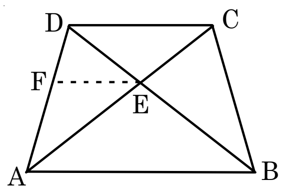
$$\frac{\text{Area of }(\Delta ADE)}{\text{Area of }(\Delta BDE)} = \frac{AE}{EC} \qquad ...(4)$$

From (1) and (4), we get $\frac{AD}{DB} = \frac{AE}{EC}$ **Q.E.D.**

Second Part :

Let ABCD be a trapezium in which diagonals AC and BD intersect at E.

To prove : $\frac{DE}{EB} = \frac{CE}{EA}$



Construction : Draw FE || AB.

Proof : In ΔABD, FE || AB

$$\Rightarrow \qquad \frac{DE}{EB} = \frac{DF}{FA} \qquad ....(1)$$

In ΔCDA, FE || DC

$$\Rightarrow \qquad \frac{CE}{EA} = \frac{DF}{FA} \qquad .....(2)$$

From (1) and (2) $\Rightarrow \frac{DE}{EB} = \frac{CE}{EA}$

Thus, diagonals of a trapezium cut each other proportionally. **Q.E.D.**

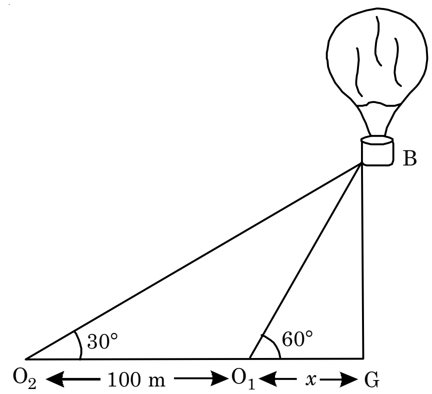
34. One observer estimates the angle of elevation to the basket of a hot air balloon to be 60°, while another observer 100 m away estimates the angle of elevation to be 30°. Find :

(*a*) The height of the basket from the ground.

(*b*) The distance of the basket from the first observer's eye.

(*c*) The horizontal distance of the second observer from the basket.

Sol.



Let 'B' be basket of hot air balloon and O_1, O_2 be first and second observer respectively.

Angle of elevations are $\angle BO_1G = 60°$ and $\angle BO_2G = 30°$.

$$O_1O_2 = 100 \text{ m}$$

Let $O_1G = x$ m

(*a*) In rt. $\angle d$ ΔBGO_1,

$$\tan 60° = \frac{BG}{O_1G}$$

$$\Rightarrow \quad \sqrt{3} = \frac{BG}{x} \Rightarrow BG = x\sqrt{3} \text{ m} \qquad ....(i)$$

In rt. $\angle d$ ΔBGO_2,

$$\tan 30° = \frac{BG}{O_2G} \Rightarrow \frac{1}{\sqrt{3}} = \frac{BG}{(x+100)}$$

$$\Rightarrow \qquad BG = \left(\frac{x+100}{\sqrt{3}}\right) \qquad ...(ii)$$

Equating (*i*) and (*ii*),

$$x\sqrt{3} = \frac{x+100}{\sqrt{3}}$$

$\Rightarrow 3x = x + 100 \Rightarrow 2x = 100 \Rightarrow x = 50$ m.

$$O_1G = 50 \text{ m} \qquad ...(iii)$$

Putting value of x in (*i*),

$$BG = 50\sqrt{3} \text{ m} \qquad ...(iv)$$

So, height of basket from ground is $50\sqrt{3}$ m.

(*b*) Again in rt. $\angle d$ ΔBGO_1, $\sin 60° = \frac{BG}{O_1G}$

$$\Rightarrow \qquad \frac{\sqrt{3}}{2} = \frac{50\sqrt{3}}{O_1B} \qquad \text{(by using } (iv))$$

$$\Rightarrow \qquad O_1B = 100 \text{ m.}$$

So, distance of first observer's eye from basket is 100 m.

(*c*) Horizontal distance of second observer from the basket = O_2G

$= O_2O_1 + O_1G$

$= 100 + x = 100 + 50$ (by using (*iii*))

$= 150$ m

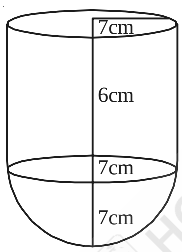
35. A vessel is in the form of a hollow hemisphere mounted by a hallow cylinder. The diameter of the hemisphere is 14 cm and the total height of the vessel is 13 cm. Find the inner surface area and the volume of the vessel.

Or

A solid toy is in the form of a hemisphere surmounted by a right circular cone of same radius. The height of the cone is 10 cm and the radius of the base is 7 cm. Determine the volume of the toy. Also, find the area of the coloured sheet required to cover the toy.

$$\left(\textbf{Use } \pi = \frac{22}{7} \textbf{ and } \sqrt{149} = 12.2\right)$$

Sol. Here, Diameter of hemisphere = Diameter of cylinder = 14 cm.



$\therefore$ Radius of hemisphere = Radius of cylinder = 7 cm

Height of vessel = 13 cm

$\therefore$ Height of cylinder = 13 – 7 = 6 cm

$\therefore$ Inner surface area of vessel = Curved Surface area of cylindr + curved surface area of hemisphere

$$2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$$

$$= 2 \times \frac{22}{7} \times 7\,(6 + 7)$$

$$= 2 \times \frac{22}{7} \times 7 \times 13 = 572 \text{ cm}^2$$

Volume of vessel $= \pi r^2 h + \frac{2}{3}\pi r^3$

$$= \pi r^2\left(h + \frac{2}{3}r\right)$$

$$= \frac{22}{7} \times 7 \times 7\left(6 + \frac{2}{3} \times 7\right)$$

$$= 22 \times 7\left(\frac{18 + 14}{3}\right)$$

$$= 154 \times \frac{32}{3}$$

$$= \frac{4928}{3} = 1642.67 \text{ cm}^3.$$

Or

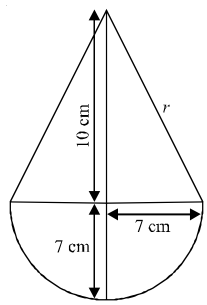
Sol. Height of cone (h) = 10 cm

Radius of cone (r)

= Radius of hemisphere = 7 cm

$\therefore$ Slant height (l) of cone $= \sqrt{h^2 + r^2}$

$= \sqrt{(10)^2 + 7^2} = \sqrt{149} = 12.2$ cm



Vol. of toy = Vol. of cone + Vol. of hemisphere

$$= \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

$$= \frac{\pi r^2}{3}[h + 2r]$$

$$= \frac{22 \times 7 \times 7}{7 \times 3}[10 + 2 \times 7]$$

$$= \frac{22 \times 7}{3} \times (24) = 1232 \text{ cm}^3.$$

Area of coloured sheet required to cover the toy

= Lateral S.A. of cone + Lateral S.A. of hemisphere

$$= \pi rl + 2\pi r^2 = \pi r\,(l + 2r)$$

$$= \frac{22 \times 7}{7}(12.2 + 2 \times 7)$$

$$= 22 \times (26.2) = 576.4 \text{ cm}^2$$

SECTION—E

This section consists of 3 Case-Study Based Questions of 4 marks each.

36. The figure below shows the path of a diver, when she takes a jump from the diving board. Clearly it is a parabola.

Annie was standing on a diving board, 48 feet above the water level. She took a dive into the pool. Her height (in feet) above the water level at any time '*t*' in seconds is given by the polynomial *h* (*t*) such that *h* (*t*) = $-16t^2 + 8t + k$.

Answer the following questions :

(*i*) What is the value of k ?

(*ii*) At what time will she touch the water in the pool ?

Or

The zeroes of the polynomial r (t) = $-12t^2 + (k - 3)\,t + 48$ are negative of each other. Then find the value of k.

(*iii*) Another diver Rita whose height (in feet) above the water level is given by polynomial p (t) with zeroes – 1 and 2. Then write the polynomial p (t).

Sol. **(*i*)** Initially, at $t = 0$. Annie's height is 48 ft

So, at $t = 0$, h should be equal to 48

$h(0) = -16(0)^2 + 8(0) + k = 48$

So, $k = 48$.

(*ii*) When Annie touches the pool, her height = 0 feet

i.e. $-16t^2 + 8t + 48 = 0$ (above water level)

$\Rightarrow \quad 2t^2 - t - 6 = 0$

$\Rightarrow \quad 2t^2 - 4t + 3t - 6 = 0$

$\Rightarrow 2t(t-2) + 3(t-2) = 0$

$\Rightarrow \quad (2t+3)(t-2) = 0$

i.e. $\quad t = 2$ or $t = -\frac{3}{2}$

Since time cannot be negative, so $t = 2$ seconds.

Or

Sol. When the zeroes are negative of each other, sum of the zeroes = 0

So, $\quad \frac{-b}{a} = 0$

$\Rightarrow \quad -\frac{(k-3)}{-12} = 0$

$\Rightarrow \quad \frac{k-3}{12} = 0$

$\Rightarrow \quad k - 3 = 0$, *i.e.* $k = 3$.

(*iii*) $t = -1$ and $t = 2$ are the two zeroes of the polynomial $p(t)$

Then $\quad p(t) = k(t+1)(t-2) = k(t^2 - t - 2)$

When $\quad t = 0$ (initially) $h_1 = 48$ ft

$p(0) = k(0^2 - 0 - 2) = 48$

i.e. $\quad -2k = 48$

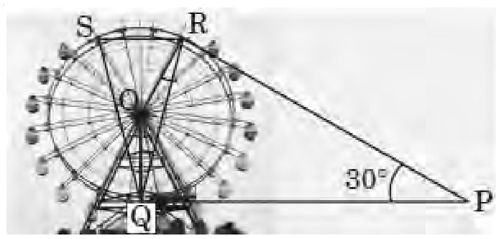
$\Rightarrow \quad k = -24$

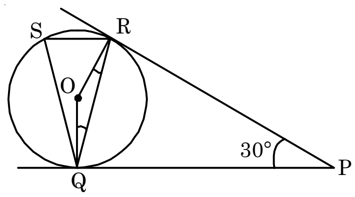
So, the polynomial is $-24(t^2 - t - 2)$

$= -24t^2 + 24t + 48$.

37. A Ferris wheel (or a big wheel in the United Kingdom) is an amusement ride consisting of a rotating upright wheel with multiple passenger- carrying components (commonly referred to as passenger cars, cabins, tubs, capsules, gondolas or pods) attached to the rim in such a way that as the wheel turns, they are kept upright usually by gravity.

After taking a ride in Ferris wheel. Aarti came out from the crowd and was observing here friends who were enjoying the ride. She was curious about the different angles and measures that the wheel will form. She forms the figure as given ahead :





Answer the following questions :

(*i*) In the given figure, find the value of ∠ROQ.

(*ii*) In the given figure, find the value of ∠RSQ.

(*iii*) In the given figure, find the value of ∠RQP.

OR

In the given figure, find the value of ∠QRP.

Sol. In quadrilateral PQOR,

$\angle ORP + \angle PQO + \angle RPQ + \angle ROQ = 360°$

$\Rightarrow 90° + 90° + 30° + \angle ROQ = 360°$

[∵ Radius ⊥ Tangent]

$\Rightarrow \quad \angle ROQ = 150°$

(*ii*) As angle subtended by an arc at the centre of circle is twice the angle it subtends anywhere on the circle's circumference.

$\therefore \quad \angle QOR = 2\angle RSQ$

$\Rightarrow \quad \angle RSQ = \frac{1}{2}\angle QOR = \frac{1}{2}(150°) = 75°$

(*iii*) ΔORQ is isosceles triangle.

$\Rightarrow \angle ORQ = \angle OQR$

(∵ OR = OQ = Radii of same circle)

$\Rightarrow \angle ORQ + \angle RQO + \angle QOR = 180°$

$\Rightarrow \quad 2\angle RQO + 150° = 180°$

$\Rightarrow \quad \angle RQO = \frac{30°}{2} = 15°$

$\angle RQP = \angle OQP - \angle RQO$

$= 90° - 15° = 75°$

Or

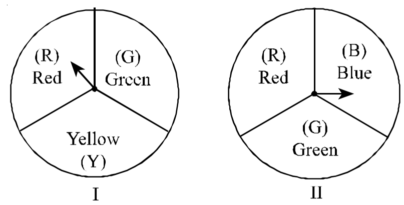
Radius ⊥ Tangent at the point of contact.

$\therefore \quad \angle ORP = 90°$

$\therefore \quad \angle QRP = \angle ORP - \angle ORQ$

$= 90° - 15° = 75°.$

38. A middle school decided to run the following spinner game as a fund-raiser on Christmas Carnival.



Making Purple: Spin each spinner once. Blue and red make purple. So, if one spinner shows Red (R) and another Blue (B), then you 'win'. One such outcome is written as 'RB'. **(*CBSE 2023*)**

Based on the above, answer the following questions:

(*i*) List all possible outcomes of the game.

(*ii*) Find the probability of 'Making Purple'.

(*iii*) For each win, a participant gets ₹ 10, but if he/she loses, he/she has to pay ₹ 5 to the school. If 99 participants played, calculate how much fund could the school have collected.

Or

If the same amount of ₹ 5 has been decided for winning or losing the game, then how much fund had been collected by school ? (Number of participants = 99)

Sol. (*i*) All possible outcomes are S = {RR, RB, RG, GR, GB, GG, YR, YB, YG)

(*ii*) P (making purple) = P (RB) = $\frac{n(\{RB\})}{n(S)} = \frac{1}{9}$.

(*iii*) Tatal fund = (number of participants) × {P(win) × ₹ 10 – P(loss) × ₹ 5}

$= 99 \times \left[\frac{1}{9} \times 10 - \frac{8}{9} \times 5\right]$

[$\therefore$ P (loss) = 1 – P (win) = $1 - \frac{1}{9} = -\frac{8}{9}$]

$= 99 \times \left[\frac{-30}{9}\right] = -$ ₹ 330

[The negative sign indicates participant would give ₹ 330 to school]

∴ School could have collected ₹ 330 from the game.

Or

Total fund of ₹ 5 has been decided for winning as well as losing

Total fund = $99 \times \left\{\frac{1}{9} \times 5 - \frac{8}{9} \times 5\right\} = 99 \times \frac{-35}{9}$

= – ₹ 385

[The negative sign indicates participants would end up losing amount to school]

So school would have collected ₹ 385 from participants.